THE PRINCIPLE OF RELATIVITY

IN THE

LIGHT OF THE PHILOSOPHY OF SCIENCE

The Principle of Relativity

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Light of the Philosophy of Science

By

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With an Appendix Containing a Letter from the Rev. James Bradley on the Motion of the Fixed Stars, 1727

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THE PRINCIPLE OF RELATIVITY.

INTRODUCTORY.

PHYSICAL science seems to have entered into a new phase, the slogan of the new school being THE PRINCIPLE OF RELATIVITY. In some quarters the current modes of thought are declared antiquated, and the promise is made that the old truths will acquire a new meaning. Physicists speak of the relativity of time and space, and we will add that they ought as well speak of the relativity of things, of the whole actual world in all its parts and interrelations.

Many who have watched the origin and rise of the new movement are startled at the paradoxical statements which some prominent physicists have made, and it is remarkable that the most materialistic sciences, mechanics and physics, seem to surround us with a mist of mysticism. The old self-contradictory statements of the Eleatic school revive in a modernized form, and common sense is baffled in its attempt to understand how the same thing may be longer and shorter at the same time, how a clock will strike the hour later or sooner according to the point of view from which it is watched; and the answer of this most recent conception of physics to the question, How is this all possible? is based on the principle of the relativity of time and space.

The man who started this movement and was the first to formulate it in concise language and to base it upon close

argument was Professor Einstein,[1] who was followed by Lorentz,[2] and so we hear often of the Einstein-Lorentz theory. The strangest thing about it is that the question is seriously debated whether or not this theory is true, and the answer is expected from experiments; while in our opinion we are here confronted with a method, and the problem is simply how we can best deal with certain difficulties due to the relativity of all things. These difficulties have originated through the need of a greater exactness in measurements, but the underlying truth—the relativity of all things—is not a question of fact, but a recognition of certain complications with which we must learn to deal.

On reading recent expositions of the principle of relativity the man of good education, or the one who has attended universities without being a specialist in either mathematics or physics, feels the *terra firma* give way under his feet, and when he finds that the principle of identity seems to fail in his comprehension of things, a dizziness comes over his intellect and he sinks into the bottomless abyss of the incomprehensibility of existence. A general earthquake seems to quiver through his mind. Everything totters around him and he stands in awe at the significance of the new thought. Nor is there any one who dares to contradict; for the most learned arguments are adduced, the mathematical and logical conclusions of which bristle with formidable formulas,—yea, experiments are made to prove the truth of the relativity of time and space.

For the sake of convenience we will speak of the representatives of this new conception as the "relativity physicists" in contradistinction to the old-fashioned physicists of the old school. It has been said that the former represent more the mathematical aspect of physics while the

[1] *Jahrbuch der Radioaktivität und Elektronik*, 1905-1908.

[2] H. A. Lorentz, *Theory of Electrons* (Teubner) 1910.

latter are the realistic physicists proper, too realistic to understand the significance of the new truth.

In order to facilitate a comprehension of the situation as well as our own conception, we will here at once and dogmatically state that the relativity physicists are perfectly right; what they claim is really and truly a matter of course, and if they only would present their proposition without dressing up their theory in paradoxical statements, nobody would in the least hesitate to accept the new view. But as soon as this is done people will at the same time find out that the new view is not novel. Its importance has been greatly exaggerated, for the principle has been tacitly understood in the correct way by all preceding physicists who, at the time however, ignored, or better did not enter into, the problem, because they had other more pressing work on hand. Nor is it unlikely that they regarded this problem of relativity as a philosophical question which strictly speaking had no place before the forum of physics.

ON THE ABSOLUTE.

Perhaps the easiest way of elucidating the true meaning of the relativity of time and space will be by setting forth our own position as we held it long before the principle of relativity gained prominence or had even been mentioned or alluded to.

The writer's book *Fundamental Problems* contains the following statement under "Definitions and Explanations" (first edition, page 254; seecond edition, page 252):

"Absolute existence (in fact everything absolute) is impossible. Reality is properly called *Wirklichkeit* in German, derived from *wirken,* to take effect. Reality is not immovable and unchangeable absoluteness, but the effectiveness of things in their relations. Reality therefore implies not only existence, but the manifestation of existence

also. Existence and its manifestation are not two different things; both are one."

Since the days of Heraclitus it has been a trite truism that all existence is in a flux. There is no rest anywhere, and actuality consists in the effects which these changes exercise upon one another by action and reaction. Upon this lack of stability, resulting from a universal and intrinsic relativity, Mr. Spencer bases one of the strongest, though quite untenable, arguments of his agnosticism. He seems to expect that time, space, motion, and matter are or should be things-in-themselves, and forgets that they represent relations, i. e., certain features of reality. We will here quote his exposition of the unknowableness of motion in space. In his *First Principles* Spencer says:

"Here, for instance, is a ship which, for simplicity's sake, we will suppose to be anchored at the equator with her head to the west. When the captain walks from stem to stern, in what direction does he move? East, is the obvious answer,—an answer which for the moment may pass without criticism. But now the anchor is heaved, and the vessel sails to the west with a velocity equal to that at which the captain walks. In what direction does he now move when he goes from stem to stern? You cannot say east, for the vessel is carrying him as fast towards the west as he walks to the east; and you cannot say west for the converse reason. In respect to surrounding space he is stationary; though to all on board the ship he seems to be moving. But now are we quite sure of this conclusion? Is he really stationary? When we take into account the earth's motion round its axis, we find that instead of being stationary he is traveling at the rate of 1000 miles per hour to the east; so that neither the perception of one who looks at him, nor the inference of one who allows for the ship's motion, is anything like the truth. Nor indeed, on further consideration, shall we find this revised conclusion to be much better. For we have forgotten to allow for the earth's motion in its orbit. This being some 68,000 miles per hour it follows that, assuming the time to be midday, he is moving, not at the rate of 1000 miles per hour to the east, but at the rate of 67,000 miles per hour to the west. Nay, not even now have we discovered the true rate and the true direction of his movement. With the

earth's progress in its orbit, we have to join that of the whole solar system towards the constellation of Hercules; and when we do this, we perceive that he is moving neither east nor west, but in a line inclined to the plane of the ecliptic, and at a velocity greater or less (according to the time of the year) than that above named. To which let us add, that were the dynamic arrangements of our sidereal system fully known to us, we should probably discover the direction and rate of his actual movement to differ considerably even from these. How illusive are our ideas of motion, is thus made sufficiently manifest. That which seems moving proves to be stationary; that which seems stationary proves to be moving; while that which we conclude to be going rapidly in one direction, turns out to be going much more rapidly in the opposite direction. And so we are taught that what we are conscious of is not the real motion of any object, either in its rate or direction; but merely its motion as measured from an assigned position—either the position we ourselves occupy or some other."

The same argument of the captain walking the deck of a ship was made before Spencer, though mostly it was a ball rolling on deck; Bradley refers to it as well known in his time, 1727, and the same story has been repeated after Spencer. In fact it is one of the arguments of the relativity of space among modern relativity physicists.

The principle upon which the representatives of the new view take their stand is a consideration of actual life. Things are in a flux, and this is an undeniable fact. We must bear in mind that the way of making knowledge possible at all in the flux of being is to ignore what has nothing to do with the problem under investigation. Our method is based upon a fiction or, if you please, upon an artificial trick, viz., to ignore complications and to consider a certain thing as fixed; but there are cases in which we must remember that we ourselves change and that the very position we assume is moving.

This trick of assuming that our position is stable is easy enough because man does not at once notice that there is any change; but all things are in a flux and he himself

changes unconsciously. A primitive unsophisticated man does not know that the earth on which he stands is whirling around itself at the rate of 1037 miles an hour, on the equator, further that it is also revolving with incredible speed around the sun, and that with the sun it is proceeding in a spiral motion towards one of the constellations, probably the constellation Heracles, around an unknown center situated somewhere in the Milky Way. God only knows what else takes place and what kind of whirling dances the Milky Way performs. The savage has not the slightest idea of all this, and so it is easy for him to ignore the motion of which he unconsciously partakes.

If man really were aware of all the events which influence him, his head would swim, and he would be incapable of thinking any sober thought. Fortunately he is concerned solely with his own narrow interests. The more man in the further growth of his mind becomes familiar with these unnoticeable events, the more he discovers that for any particular purpose he must ignore what does not belong to the solution of the special problem under consideration.

This way of ignoring what does not concern us at the time is an artificial process, a process of abstraction and elimination, of cutting off all disturbing incidents, and in doing so the philosophically minded scientist will become aware of the fiction of arbitrarily laying down a point of reference which is treated as if it were stable while in fact, like everything else, it too is caught in the maelstrom of cosmic existence.

There is nothing wrong or harmful in this fiction; on the contrary it is an indispensable part of our method of comprehending things. The universe is too complicated to be understood or viewed at a glance, and knowledge, science, cognition as well as all mental processes become possible merely by concentration, i. e., by selecting a point of

view as being a certain fixed location from which we observe a change, an event, a transformation, in order to gain a comprehension of this or that piece of existence in contrast to others of the same or of a different kind. Such is the nature of cognition, and this artificial trick is an essential condition of observation.

Knowledge is relative. It is the relation between subject and object, the thinker and the thing, and this, far from being objectionable, is only the universal condition of all existence; for all existence is relative. All reality is the result of action and reaction; it is a forming and being formed under definite conditions; it is transformation. There is no existence in and by itself. Relativity is the principle of all real and actual being.

TRICKS OF COGNITION.

If the standpoint of an observer changes, the thing observed will naturally change too in its relation to him. Formerly physicists were in the habit of not seriously bearing in mind that the fixedness of their standpoint was an assumption; they did not follow this principle to its ultimate consequences. For their special problems it was not necessary to do so, and there is very little use in bearing it constantly in mind. The difference in time between the moment when the observer looks at an object and that in which the rays of light indispensable for observation strike his eye is too inconsiderable to be taken into account; it is a negligible quantity. But if the object under consideration is at such an enormous distance that it takes the rays of light thousands of years to reach the eye of the astronomer it does make a difference, and so James Bradley was astonished to register the fact that the fixed stars in the sky were not always in the same place but that they pendulated semi-annually above us with the motion of the earth around the sun. The direction in which we see them swings from

the aphelion to the perihelion, and a closer consideration of the facts shows that the rays of very distant stars which we catch in the aphelion are not caused at the moment when we see them but started thousands of years prior to the moment in which they strike the lens of the astronomer's telescope, and so the transference of rays of light from the star to the astronomer's eye at this enormous distance represents a relation which most forcibly drives the truth home to us that there is nothing absolute.

The same is true of all things. The object before us seems to stand there in a perfect and quiet completeness, and yet the changes that work unnoticed by our dull senses are constant, continuous and rapid. Heraclitus used to say that he could not come out of the same river into which he had stepped a moment before, because the water was always rushing by. Never is a drop of it the same, and this is true of all things, even of ourselves. The observer has to exclude from his methods of observation the fact that he himself, his senses and his mind, are in a constant flux.

In order to elucidate the significance of the nature of cognition as being a limitation and concentration upon one point and constructing artificial units, the writer has on former occasions used the analogy of the kinematoscope, the machine which produces moving pictures.

In order to make any picture possible we need a lens, and the lens focuses the rays of light so as to throw rays from the same spot upon one and the same place on the plane where the picture appears. The rays of light which proceed from an object scatter in all directions, and unless we use a lens to concentrate the rays, the formation of a picture of the object would remain impossible. Thus the method of producing a picture is by concentration.

The lens produces a picture by focusing rays of light, that is by throwing the same rays upon the same spot; but it would also be possible to produce a picture by cutting

off the redundant rays of light and singling out one or very few rays, each one coming from each of the several points of the object. Accordingly we can photograph objects through a pinhole; there is only this difference that the picture is weak and needs long exposure. This proves that the process of concentration is fundamentally a process of abstraction, of leaving out, of omitting the disturbing multiplicity of the innumerable facts of real life as represented in the totality of objective experience.

The kinematoscope involves not only the static form of things, their spatial expression, the juxtaposition of parts, but it also adds the changes that are taking place in time. The film of the kinematoscope consists of a series of pictures, one always a little different from another, and if these are presented in rapid succession the series is fused into one picture in which the succeeding differences appear as motion. This is accomplished by the introduction of a little winged wheel which in rapid succession covers and uncovers the several pictures. If we would take this little wheel with its wings out of the kinematoscope, and if otherwise the pictures on the film would succeed one another in a rapid continuous motion without this artificial separation by the wings of the wheel, we would see no picture at all but simply have a blur on the canvas. In order to have distinct pictures appear on the canvas, we must cut the flux of motion into little separate moments which we may allegorically characterize as atoms of time.

Reality is a continuous flux, but in order to follow it step by step we must do the same thing that the mathematician does with his differential calculus. In the calculus the curve is cut up into infinitesimal lines, which in continuous succession change their directions, and the smaller we conceive these lines to be, the less is the mistake made by this fiction, if they are treated like straight lines.

The method of the calculus, based upon the fiction of

substituting for a continuous curve a series of little straight lines constantly changing their direction, is not so very different from the method of cognition in general. Nor is there anything wrong in it, only we must remain conscious of the fiction. In a similar way we must know that existence itself is a continuous system of relations, or in other words, that relativity is the principle of all existence in the world of actual life as well as in the domain of thought. We must cut up the general flux according to the needs of our investigation and lay down artificial limits.

* * *

If we view the new physics under this aspect, it will lose its mystic glamor and at the same time appear intelligible. In fact we shall understand that the principle of relativity is a matter of course, and if we cut up reality into things, as if they were things-in-themselves, into units or atoms, we employ a trick of cognition which makes it possible to focus things and picture them distinctly in our mind.

There are large numbers of scientists possessed of an *odium philosophicum* because philosophy means to them some abstruse metaphysical system of thought which ignores the natural sciences and, spiderlike, spins a world-conception out of pure thought derived from the thinker's subjectivity. The result is that they are soon perplexed in their own science by philosophical problems; for true philosophy—the philosophy of science—is an indispensable factor of cognition, and its influence extends into the fabric of all scientific labors. Thus it happens that problems of a philosophical character arise unexpectedly, and then the information given by nature in reply to experiments is apt to be misunderstood.

If the reference point (R) from which an observer measures is in motion toward R_1, and the object observed (O) also possesses a motion of its own, we are confronted

with a complicated phenomenon. If R moves toward O, the object measured will be shorter than if it stands still, and it will be longer if R moves with O in the same direction. We have only to forget, after the fashion of the pragmatist, that there is an ideal of objective cognition, and assume that all there is about size or the objective measure of things consists in the result of our measuring and we have the clue to the paradoxes of the physics of relativity. If the point of reference is not stationary and if we neglect to account for its motion, the result of our measurement is necessarily vitiated thereby as much as the pragmatist's philosophy by his personal equation.

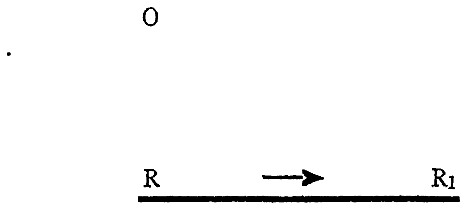


Fig. 1.

There are further complications of measurement. The time needed for the transmission of signals must also be taken into consideration. The rays of light travel at an enormous velocity but the distances in the starry heavens are also enormous and the distance between O and R is less than between O and R_1. The rays which were sent out from O at the moment of measurement have already passed the track of the observer at R, while this same observer has moved on to R_1, and there he catches the rays sent out from O in its position at O; in the meantime however the object O has in its turn also changed its place. From R_1 it appears at O, where it stood while the observer was stationed at R, but in fact it stands no longer at O but has in the meantime proceeded on its own path whithersoever that may have led O, backward or forward, in any

other direction than R, possibly in the same direction as R. Such phenomena are necessary results of the relativity of existence, and we must bear them in mind when confronted with complicated conditions which present themselves, for instance in astronomical cases. Here the mistakes rising from the fiction of assuming our reference point to be stable are considerable enough to enforce attention, and in that case we shall have to make allowance for the instability of our reference point, as well as for the time which the rays of light need for their travel through space.

That was exactly Bradley's case as set forth in his essay written in 1727, one hundred and eighty-five years ago, and thus he became the forerunner of the relativity physicists. To state it in other terms, Bradley correctly solved a problem which in our days led to the formulation of the principle of relativity, and he did so without mentioning this theory, yea without feeling the need of formulating it. He simply took it for granted that he had in this case to consider the motion of the earth that served him as a reference point—the place of his observations.

COMSTOCK ON RELATIVITY.

The most popular and at the same time the most exact characterization of the principle of relativity comes from the pen of Prof. D. F. Comstock, of the Massachusetts Institute of Technology. It appeared in *Science* (Vol. XXXI, 1909, p. 767), and we quote from it the passages which contain the statement of the problem:

Professor Comstock starts with the following two postulates:

> "The uniform translatory motion of any system can not be detected by an observer traveling with the system and making observations on it alone.
>
> "The velocity of light is independent of the relative velocity of the source of light and observer."

The main passages of his exposition state the problem thus:

"The whole principle of relativity may be based on an answer to the question: When are two events which happen at some distance from each other to be considered simultaneous? The answer, 'When they happen at the same time,' only shifts the problem. The question is, how can we make two events happen at the same time when there is a considerable distance between them.

"Most people will, I think, agree that one of the very best practical and simple ways would be to send a signal to each point from a point half-way between them. The velocity with which signals travel through space is of course the characteristic 'space velocity,' the velocity of light.

"Two clocks, one at A and the other at B, can therefore be set running in unison by means of a light signal sent to each from a place midway between them.

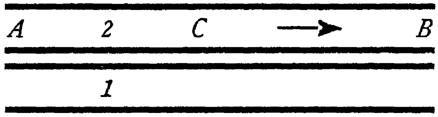


Fig. 2.

"Now suppose both clock A and clock B are on a kind of sidewalk or platform moving uniformly past us with velocity v. In Fig. 2 (2) is the moving platform and (1) is the fixed one, on which we consider ourselves placed. Since the observer on platform (2) is moving uniformly he can have no reason to consider himself moving at all, and he will use just the method we have indicated to set his two clocks A and B in unison. He will send a light flash from C, the point midway between A and B, and when this flash reaches the two clocks he will start them with the same reading.

"To us on the fixed platform, however, it will of course be evident that the clock B is really a little behind clock A, for, since the whole system is moving in the direction of the arrow, light will take longer to go from C to B than from C to A. Thus the clock on the moving platform which leads the other will be behind in time.

"Now it is very important to see that the two clocks are in unison for the observer moving with them (in the only sense in which the word 'unison' has any meaning for him), for if we adopt the first

postulate of relativity, there is no way in which he can know that he is moving. In other words, he has just as much fundamental right to consider himself stationary as we have to consider ourselves stationary, and therefore just as much right to apply the midway signal method to set his clocks in unison as we have in the setting of our 'stationary clocks.' 'Stationary,' is, therefore, a relative term and anything which we can say about the moving system dependent on its motion, can with absolutely equal right be said by the moving observer about our system.

"We are, therefore, forced to the conclusion that, unless we discard one of the two relativity postulates, the simultaneity of two distant events means a different thing to two different observers if they are moving with respect to each other."

We quote further:

"It must be emphasized that, because of the first fundamental postulate, there is no universal standard to be applied in settling such a difference of opinion. Neither the standpoint of the 'moving' observer nor our standpoint is wrong. The two merely represent two different sides of reality. Any one could ask: What is the 'true' length of a metal rod? Two observers working at different temperatures come to different conclusions as to the 'true length.' Both are right. It depends on what is meant by 'true.' Again, asking a question which might have been asked centuries ago, is a man walking toward the stern of an eastbound ship really moving west? We must answer 'That depends' and we must have knowledge of the questioner's view-point before we can answer yes or no."

The question of the man walking on a ship not only "might have been asked centuries ago," but it has been asked centuries ago. Our forebears were more conscious of the relativity of existence than the relativity physicists credit them.

Professor Comstock continues:

"It must be remembered that the results of the principle of relativity are as true and no truer than its postulates. If future experience bears out these postulates then the length of the body, even of a geometrical line, in fact the very meaning of 'length,' depends on the point of view, that is, on the relative motion of the observer and the object measured."

Professor Comstock's verdict of the case is summarized in this paragraph:

"The results of the principle for uniform translation are simply as true as its two postulates. If either of these postulates be proved false in the future, then the structure erected can not be true in its present form. The question is, therefore, an experimental one."

Here we demur. We claim that the question is not experimental but belongs to the department of *a priori* reasoning.

Professor Comstock does not enter into questions of mass connected with the principle of relativity but is satisfied with this comment:

"The apparent transverse mass is, I think, best derived by Lewis and Tolman,[3] in their excellent paper on the principle of relativity, and the relation between transverse and longitudinal mass is shown in the most direct and simple way by Bumstead[4] making use of the torsion pendulum. Any one interested in the subject should read these two papers."

THE A PRIORI.

It is characteristic of modern science to denounce the principle of the *a priori* and to extol experiment and experience. Now it is true that experience and experiment are indispensable factors in science, and in all the specialties of science. In experience and experiments we deal with the facts presented to us by nature; but the method of reasoning is not a thing which is derived from sense experience.

The method of reasoning is, as Kant truly said, *a priori* and, let us add, the *a priori* is nothing mystical or mysterious; it is simply the result of pure thought or reflection from which the data of the senses have been excluded. Pure thought (or better, purely formal thought) is a mental construction, or, if you prefer, a fiction. We omit every-

[3] *Phil. Mag.*, 18, 510-523, 1909.

[4] *Am. Jour. of Science*, *26*, pp. 493-508, 1909.

thing concrete and thus we retain a field of abstract possibilities. Elsewhere we have called it a field of anyness.[5] Obliterating in our mind all particularity we retain nothing concrete and in this field of nothingness we build up pure relations. From this domain all real things, comprising everything which we subsume under the categories of matter and energy, has been excluded. But these pure relations, i. e., pure forms which are non-material constructions lacking all concrete qualities such as all real things possess, serve us as models for the relations of any possible purely mental or actual existence. Our doings in this field of abstraction consist in the fiction of pure lines, pure numbers, pure motion, pure ideas and their interrelations such as genera and species, and thus we are capable of building up a world of purely formal or relational thought, the totality of which in space is called geometry, and in the domain of numbers which originate by counting a series of single units, arithmetic, etc. In the domain of pure thought, consisting of genera and species, we call the laws that govern their relations logic, and the law of transformation, of which the positive aspect is properly called causality, and its negative counterpart the law of conservation of matter and energy, has been called by Kant pure natural science.

All systems of mental constructions have the advantage of picturing in our mind *any* possible configuration of relativity, and in this sense pure thought (Kant's *a priori*) is a field of anyness. It can be applied to any fact or set of facts of existence, actual or fictitious, and these systems of mental constructions therefore furnish us with the key to determine the relations of real nature. They render possible the systematization of sense impressions and thus

[5] See *Philosophy of Form*, the chapter on "The Foundation of Mathematics and Logic," pp. 7-10. For further details see also the chapter "Form and Formal Thought" in the author's *Fundamental Problems*, pp. 26-60.

these systems of pure thought in the field of anyness are the methods of scientific operation.

Let us not therefore speak contemptuously of the *a priori*, or denounce apriorism as something medieval and elusive, for even here in the attempt at establishing the principle of relativity in time and space, the arguments of the physicists are absolutely aprioristic. There is not one of these so-called experiments, invented to prove the relativity of time and space, which does not ultimately resolve itself into a machine that renders visible aprioristic considerations.

The ultimate arguments in all the experiments made to prove the relativity of time and space move in a domain of purely formal thought, and the force of them is ultimately of the same kind as the Q. E. D. of Euclidean theorems. We think here mainly of such propositions as locate an observer on the sun and another on the earth. Their clocks actually agree, but when compared they are found to differ. About eight minutes have elapsed when the observer on earth registers the time as the rays of the sun reach the earth, and *vice versa* when the clock on earth is observed as the rays from the earth strike the sun. The imitation of the same conditions for the sake of comparing the registration of two moving systems in an actual experiment amounts to nothing more than the pencil drawings of a Euclidean or logical figure in which the *a priori* reasoning is visibly presented as a *demonstratio ad oculos.* The argument remains in either case one of pure thought.

The photograph of such an apparatus built for the purpose of making an experiment in the relativity of time and space to show the difference between a solar clock and a terrestrial clock may be found in the article of Emil Cohn of Strassburg, "Physikalisches über Raum und Zeit" in *Himmel und Erde,* Vol. XXIII. To be sure the instrument does not fulfil the conditions either of distance or of

the velocity of the transference of the signal, "but," says Professor Cohn, "that is of secondary importance."

There are two motions both constant and both standing in a definite proportion. The sun with its clocks has been made to stand still. The earth with its two clocks moves, and there is an arrangement by which to represent the transference of signals. The main thing is that "their velocities stand in definite proportions and all that concerns us are these proportions. That we have here replaced the enormous velocity of light by a velocity of a few centimeters per seecond is unessential. It is essential, however, that the velocity of the earth is three-fourths the velocity of light, while the real ratio is 1: 10,000."

Newton's laws are *a priori*, and Newton proves that these laws hold good in, and are serviceable as, interpretations of the actual world of fact. The empiricist ought to rebel against Newton's laws, because they never have been nor ever can be proved by either experience or experiment. Whoever saw a body moving in a straight line? and has Newton (from the standpoint of the empiricist) any right at all to make such sweeping statements of movements which have never occurred in the experience of anybody?

The most general principle at the bottom of scientific work is perhaps the so-called law of the conservation of matter and energy, and even this law is based on purely *a priori* arguments.

Incidentally we will say that the law does not hold good if we restrict the notion of matter to matter in the sense of the physicist which is mass, i. e., to concrete particles of existence that are extended and possess weight. It holds good only if we understand by matter the substance of being, its objective reality. We had better therefore speak of the conservation not of matter but of substance, for gross matter, consisting of the chemical elements, is constantly being produced before our eyes in the starry

heavens where the astronomers can watch the process through their telescopes. In the nebulas we see now the commotion of whirls with which gradually first the lighter and then the heavier chemical elements are being manufactured out of the original world-substance which we assume to be the same as the luminiferous ether.

Therefore we may surrender the law of conservation of gross matter, but we still hold to the conception that there is a conservation of stuff or substance, and the same is true of energy. There may be energy in the shape of a stress incorporated in the same wonderful world stuff, the ether, and this stress may be set free and become actual motion or kinetic energy, by some cause which creates those whirls that start the formation of nebulas.

And what proves the law of this conservation of substance and energy? It is the necessity of *a priori* thought which compels us to assume the principle that nothing originates from nothing and nothing disappears into nothing, which thought rests ultimately on the idea that all processes of existence are transformations. Everything that originates is formed by combination from something that existed before.

It has been maintained that the principle of relativity must be proved experimentally, but this is a mistake. Reality is everywhere a system of interrelations, yea every single concrete thing, every phenomenon, every piece of existence is a bundle of relations. It can be analyzed into its elements, which are actions and reactions; and that is all that reality means. Space as well as time are merely the measures, the former of arrangement or position, the latter of succession. Space denotes the interrelation of parts constituting figures or shapes affording a mode of determining direction and distance. Time measures the duration of events which is done by counting uniform cyclical motions or parts thereof. And so we must grant

that the relativity of time and space, as well as of all real things is a universal and inalienable condition of all existence. We can not think of any actuality which would not be dominated by relativity; which means we must regard the principle of relativity as an *a priori* postulate.

The principle of relativity is not established by experience but is ultimately based upon reflection and pure ratiocination. It belongs to the category of purely formal thought as much as all arithmetical and geometrical propositions.

If any proposition of purely formal thought, such as $2 \times 2 = 4$, does not hold good in our experience, we doubt the correctness of our counting or measuring, but we do not doubt our *a priori* proposition. We revise our observation, not our logic, our arithmetic, our mathematics; and suppose our observation proves true, suppose that 2×2 rabbits shut up in a cage are on recounting their number found to be more than four, say six or ten or any higher amount, we do not upset our arithmetic or any of our purely formal propositions, but seek the cause of the irregularity in the objects, in the things or animals counted. In that case we are positive that some transformation of the concrete material has set in which adds to the number to be expected according to arithmetical law.

If the reference point (R) belongs to the same system of motion as the object observed (O), our measurement will be correct and indicate the size of the object adequately. But if R moves in a direction and with a velocity of its own, different from O, the measurement will not be adequate; it will be warped in an exact proportion to the motion of R, and this rule holds good in the same way as all mathematical, logical and generally purely formal theorems.

The reliability of purely formal truths is not merely theoretical, but finds its application in practical life, in the

objective world of matter and motion, and can be verified by experience and experiment. And this is true also of the relativity of time and space.

If for instance a photographer takes the picture of a rapid express train in motion with a camera provided with a curtain shutter, the wheels will not be round but oval in the photograph, and the relativity photographer who identifies the picture with the thing, in the same way as the relativity physicist identifies the result of measuring with the objective size of the object measured, will claim that in proportion to the velocity of the train times the inverse proportion of the velocity of the slit in the curtain of the shutter, the wheels will increase their horizontal diameters and become that much more oval. Yea they will insist that the very same wheel will be at the same time in one camera, only a little more, in another one much more oval according to the quickness with which the slit of the curtain passes over the sensitive plate.

The relativity photographer will claim that the wheels in motion *are* oval while common mortals think that they only appear oval in the photograph.

Photographs do not lie; they show the objects photographed without any personal equation on the part of the photographer; their objectivity and impartiality can not be doubted, and here we see the wheels oval. They are oval, and their ovality, viz., their deviation from true circles, depends on the velocity of certain motions. An enthusiast for the principle of relativity can justly claim that every photograph of a rapid train which shows the oval form of the wheels is a successful experiment in the demonstration of the relativity of figure in space.

The truth of the principle of relativity in the domain of photography can be explained by *a priori* considerations. It is a matter of course, and if we argue the subject in our mind in pure reflection, we find out what we must expect,

and if finally we make the experiment, the principle proves true.

In the same way all the experiments made by machinery so constructed as to represent terrestrial and solar clocks or yard sticks, and to point out the unavoidable difference of measurements in both time and size resultant from their respective motions of the earth and the sun as well as the time it takes to transmit signals, are not experiments in the physicist's sense but expositions and demonstrations of purely formal truths which belong to the category of mathematics.

If the principle of relativity does not hold good in any domain of actual life, we must seek the cause in the material used and not in the principle of relativity. In other words we would be confronted with a purely physical problem which demands a physical solution, and this seems to be the case of the Fizeau experiment.

Prof. Emil Cohn, of Strassburg,[6] says:

> "It is strange that the relativity principle of mechanics does not hold good in radiation—in radiation and therewith in electrodynamics, for that the spread of radiation is an electrical process we may consider since Heinrich Hertz as an assured matter of experience. The decisive experiment which has been made by Fizeau is this: In a liquid, flowing with a uniform velocity, light is to be propagated in the direction of the current. According to the relativity principle an observer drifting in the current should find the velocity of propagation to be the same as if the liquid were at rest, and an outside observer should find the velocity of the light augmented by the full velocity of the current in the liquid. (Think, e. g., of the ball rolling on the deck of a ship in motion.) But such is not the case. There is added only a certain portion, viz., the index of refraction."

The very result of the experiment proves that one of the determinant factors is the physical property of the fluid.

When the principle of relativity is applied to positive

[6] *Loc. cit.*, p. 7.

facts we reach slippery ground, on which we must be on our guard to avoid mystification, for it would seem as if the law of the conservation of matter and energy were upset and all objectivity of scientific truth were lost. Experiments have been made to prove the principle of relativity with the result that Hupka and Bucherer,[7] the former with cathode rays, the latter with radium rays, demonstrate that mass increases with velocity as the relativity principle demands. Kaufmann, however, comes to the conclusion that there is an increase of mass but not as ought to be expected according to the principle of relativity, while Michelson and Morley demonstrate with great exactness that in spite of the motion of the earth the transmission of light is not changed at all, not within one hundred millionth of its proportion nor even a fraction thereof.

It would lead us too far to discuss the experiments made to apply the principle of relativity to physics and electrodynamics; we will only mention that (as *a priori* might be expected) they tend to corroborate its applicability in these domains.

ON ABSOLUTE MOTION.

Dr. Philipp Frank in his discussion "Does Absolute Motion Exist?"[8] declares that motion in physics always means "motion with reference to some definite body," and he recognizes that "this question is a philosophical one[9] but it is certainly not a physical question." The answer is the first Newtonian law, viz., "A body not affected by an exterior force moves in a straight line with a constant

[7] A. H. Bucherer, "Die experimentelle Bestätigung des Relativitätsprinzips" in *Annalen der Physik,* XXVIII, p. 513; "Messungen an Becquerelstrahlen" in *Physikalische Zeitschrift,* IX, pp. 755-760.

[8] "Gibt es eine absolute Bewegung?" Lecture delivered December 4, 1909, at the University of Vienna before the Philosophical Society. *Wissenschaftliche Beilage,* 1910.

[9] Dr. Frank adds here: "Perhaps the psychologist would call it a psychological one," but this would be a mistake. Psychology has nothing to do with the subject.

velocity which of course may be zero.[10] This is called the law of inertia."

If another force affects the moving body it is subject to the second law, the law of the parallelogram of forces, according to which the body will move along the diagonal of the two forces.

The following extracts translated from Dr. Frank's essay on absolute motion will prove instructive:

"The system of the fixed stars constitutes a fundamental body. Even in shooting a cannon ball towards the south we see no deviation from the law of inertia if we consider it with reference to the fixed stars. The ball remains in the same plane; but this plane does not retain the same relative position to the meridian of the earth, wherefore, of course, with reference to the earth the law of inertia is violated. On the whole it is evident that we really recover all the observed motor phenomena when we refer Newton's laws of motion to the fixed stars. Not until they are referred to the fixed stars do these laws acquire an exact sense which makes it possible to apply them to concrete conditions.

"We shall call those motions which are referred to a fundamental body 'true movements' and those related to any other body of reference 'apparent movements.' For instance the immobility of my chair is only apparent, for when referred to the fixed stars it is in motion.

"We now ask whether there are any other fundamental bodies aside from the system of the fixed stars. Obviously not any body revolving in an opposite direction to the fixed stars can be such a fundamental body, for considered with reference to such a body all rectilinear movements are curved. Therefore the law of inertia could not hold with reference to the body in question if it is valid with reference to the fixed stars. Then too a fundamental body can possess no acceleration with reference to the fixed stars, because otherwise there would be no uniformity of the motion of inertia with reference to it. However, these conditions are not only necessary but they are sufficient to characterize a fundamental body. All bodies moving uniformly and in a straight line with reference to the fixed stars will also be fundamental bodies inasmuch as rectilinearity and

[10] The original reads thus: "Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum nisi quatenus a viribus impressis cogitur statum illum mutare."

uniformity continue to hold for them, as do likewise the supplementary velocities determined by the second law. Accordingly Newton's laws do not indicate one single fundamental body, but an infinite number moving in opposite directions with a uniform and rectilinear motion.

"Hence we may well speak of 'true' in contrast to apparent rotary motion; for all bodies revolving with reference to a fundamental body revolve with reference to all other bodies. The same is true of true acceleration because an acceleration with respect to a fundamental body is also acceleration (i. e., change of velocity) with respect to all the rest. On the other hand, there is no sense in speaking of 'true' uniform rectilinear motion; for if a body possesses a uniform velocity with respect to the fixed stars, it is itself a fundamental body possessing of course with respect to itself a velocity of zero; it is at rest.

"Accordingly there is true acceleration, but not true velocity. From this is easily derived a proposition established by Newton which is called the principle of relativity of mechanics, namely that a uniform rectilinear movement of the system as a whole makes no change in the processes within the system; that is to say, we can not tell from the processes within the system what velocity the uniform rectilinear movement possesses with reference to the fixed stars. On the other hand, the rotary motion of a system has indeed an influence on the processes within the system, as for instance in the phenomena of centrifugal force; thus the earth has become flattened at its poles because of its rotation, or if I revolve a dish full of water the water will rise at the sides."

ABSOLUTE SPACE.

If we make measurements of motions which are limited to terrestrial conditions, the earth is and must be the system which, though not absolute, must for the nonce be so considered, and in that case the earth is called the fundamental or inertial body, of our measurements. But in many purely terrestrial motions we observe in very precise and exact measurements, deviations which compel us to seek for another fundamental body.

This happens in the case of the Foucault pendulum experiments and may also be observed in a cannon ball which

if shot south along the meridian will at a great distance show a deviation toward the west. Such experiments point out that the entire system of the fixed stars ought to be regarded as the fundamental body which thus would represent to us absolute space. I say here on purpose "represent to us," not "be," because we are most probably in the same predicament as persons moving in a train to whom the train and its interrelations, so long as the train does not move in a curve, represent the fundamental body or absolute space, viz., the ultimate system of reference.

It stands to reason that bodies in translation (in which the entire system as a whole moves in the same direction with the same velocity and without any internal change even of its smallest particles) behave as if they were at rest, and so the motion of a straight line cannot be observed so long as the observer remains limited to his own system. Every deviation from a straight line, however, implies a retardation on the inner side of the curve, or, what means the same, an acceleration on the outside of the curved path of motion. Accordingly all rotations bear witness to the character of their motion as appears in the Foucault pendulum experiment and in the flattening of the earth at the poles. Since further the idea of a rectilinear motion is a mere *a priori* postulate which can never be realized in actual nature, we see that every motion that takes place anywhere is affected by the totality of the universe. We must assume that its existence (the existence indeed of every particular thing or the recurrence of any event) must be understood to be a part of the whole. It bears traces of all the influences of all masses, and of all forces of the rest of the world according to the way it is interrelated with its surrounding conditions.

The fixed stars have so far proved sufficient for our terrestrial needs to serve us as a fundamental body for

calculations of a mechanical nature; but here the problem of absolute space presents itself.

We know positively that though the fixed stars are practically a fundamental body to us for mechanical measurements, they are shifting about among themselves and no more constitute something absolute than does our own earth; and yet there has risen a controversy on this subject in which Ernst Mach applies the principle of relativity throughout the universe while Prof. Alois Höfler stands up for what he calls the absolutist theory. We will hear what Dr. Frank has to say on this point:

"Is it to a certain extent accidental, or is it essential, that the tatality of the fixed stars coincides with that fundamental body in relation to which the laws of Newton hold valid? Or to put it more clearly: If the fixed stars were set violently in motion among each other and hence could no longer constitute a fixed body of reference, would the mechanical processes on earth proceed exactly as they did before? For instance, would the Foucault pendulum move just as at present, even though it now turns with the fixed stars, whereas in that case it would not be quite clear which constellation's revolution it should join?

"Were everything to remain as of old the fundamental system of reference would not be determined by the fixed stars but would only accidentally coincide with them, and would in reality be some merely ideal or yet undiscovered body. In the other case all mechanical occurrences on earth would have to be completely altered to correspond with the promiscuous movements of the fixed stars.

"It is well known that this is the view held by Ernst Mach. It alone holds with consistent firmness to physical relativism, and it alone answers the second main question of physics in the relativistic sense.

"The opposite view is represented by Alois Höfler in his studies on the current philosophy of mechanics, and lately by G. Hamel, professor of mechanics at the technical high school of Brünn, in an essay which appeared in the annual report of the German mathematical society of 1909 on 'Space, Time and Energy as *a priori* Forms of Thought.'

"Before I enter upon the controversy itself I would like further

to elucidate Mach's view by carrying out its results somewhat farther. In his well-known essay on the *History and Root of the Principle of the Conservation of Energy*[11] Mach ascribes to the distant masses in space a direct influence on the motor phenomena of the earth which supplements the influence afforded by gravitation. Of course no effect of gravitation from the fixed stars upon the earth can be observed, yet in spite of this they influence, for instance, the plane of oscillation of the Foucault pendulum because in Mach's opinion it remains parallel to them.

"The question now arises according to what general law of nature this influence operates which does not, like gravity, produce accelerations but velocities instead. Obviously this influence must be a property belonging to every mass, for according to our present conception the fixed stars of course are precisely the same sort of masses as earthly bodies.

"However, experience teaches us that terrestrial masses have no more influence on the plane of oscillation of the Foucault pendulum than has the changing position of the moon, sun and planets; but on the other hand it is exactly the most distant masses, the fixed stars, which determine its plane of oscillation. Accordingly we must either assume that the effect is directly proportional to the distance of the masses (which would be very strange indeed) or simply assume that this effect is proportional to the effective masses and independent of the distance, whence the dominant influence of the more remote, as the far greater and more numerous, bodies would naturally follow, and Mach inclines to this latter view.

"Mach's view shows most clearly in his position with regard to Newton's famous bucket experiment. In this Newton intended to show that the centrifugal force produced by a revolving body is due not to its relative but to its absolute velocity of rotation. He suspended a bucket filled with water by a vertical cord, twisted the cord quite tightly and then let it untwist itself, in this way setting the bucket to revolve rapidly. At first the water did not rotate with the bucket and therefore the bucket had a velocity of rotation with reference to the water while in the meantime the surface of the water remained undisturbed. In time, however, friction caused the water to become so affected by the rotary motion that bucket and water revolved like one homogeneous mass whereby the centrifugal

[11] Second edition, Leipsic, 1909; English translation by P. E. B. Jourdain, Chicago, 1911.

›rce caused the water to rise at the sides of the bucket and the sur-ıce became concave.

"Hence it is evident that the centrifugal force reached its great-st strength at the moment when the relative motion of the water ith respect to the bucket became zero; hence according to New-›n this force can be produced only by the absolute rotary motion of ıe water.

"To this now Mach justly protests that only the relative rotation f the water with reference to the fixed stars is to be considered, for ıis system of the fixed stars and not the bucket is the fundamental ›dy. And indeed at first the water was at rest with reference to the xed stars, but at the close of the experiment it was revolving. 'he mass of the bucket compared to the mass of the fixed stars is ı entirely negligible quantity, so that it does not depend in the :ast upon the rotation. But we can not know, adds Mach, how ıe experiment would turn out if the sides of the bucket were miles ıick; and by this he apparently means so thick that their mass would e considerable even when compared with the mass of the system of xed stars. Then indeed might the rotation of the bucket disturb ıe action of the fixed stars.

"Höfler protests, on the other hand, that a system which is ymmetrical round its axis could not according to all our experience ı mechanics produce by its rotation that sort of an effect on the ater within it.

"This also is quite true. But the effect of the masses assumed y Mach is such that it can not be expressed in our ordinary ex-eriences with mechanics except by means of the facts of the iner-a of all motion with reference to the fixed stars. New conditions uch as the rotation of an enormously thick bucket might give rise › new phenomena. If we agree with Mach's view that the rotation f the plane of the Foucault pendulum is directly produced by the ıasses of the fixed stars, we must likewise admit, in order to be con-istent, that the relative rotation of the very thick bucket might give ise to similar effects with reference to the water, as the rotation of ıe system of the fixed stars with reference to the earth to the plane f oscillation.

"Höfler expresses his contention against Mach's thesis in the ›rm of the following question: If in Galileo's time the sky had een clouded over and had never become clear again so that we ould never have been able to have taken the stars into our calcu-ıtion. would it then have been impossible to have established our

present mechanics solely by the aid of terrestrial experiments? By this question Höfler means to say that if the connection with the fixed stars were a constituent of the concept of uniform motion, we would never have been able in such an overclouded world to have established the law of inertia, for instance, whereas in reality it is clear that this would nevertheless have been possible.

"I will not dwell on the more psychological question as to whether or how easily this would have been possible, but will only consider now the logical construction of mechanics in such a darkened world on the hypothesis that easily or with difficulty in one way or another we would have attained to our present knowledge of mechanics.

"Let us for a moment imagine ourselves in such a world. Above our heads extends a uniform vault of uninterrupted gray or black. Were we to shoot projectiles toward the south we would see that they describe paths which are curved towards the west; if we started pendulums to vibrating we would see that they would revolve their planes of oscillation in mysterious periods—I say mysterious because we might perhaps be able to perceive the change of day and night as an alternation of light and darkness, but would not be able to refer it to the movements of celestial bodies. Perhaps at first we would surmise that the motion of the pendulum could be ascribed to optical influences. I would like to see placed in such a world one of the philosophers who regard the law of inertia as an *a priori* truth. In the face of these mysterious curvatures and deflections he would probably find no adherents and he would not know himself what to make of his own standpoint.

"Finally, let us assume, there arises a dauntless man, the Copernicus of this starless world, who says that all motions proceed spontaneously in a straight line, but that this straight line is not straight with reference to the earth but with respect to a purely ideal system of reference which turns in a direction opposite to that of the earth. The period of this rotation is supplied by the period of the Foucault pendulum.

"This man would of course deny physical relativism upon the earth, for in his opinion terrestrial processes would not depend only on the relative velocities of terrestrial bodies but on something else besides, viz., their velocities with respect to a purely ideal system of reference. Nevertheless, he would not introduce any non-physical element because for the purpose of the physicist a purely ideal system of reference whose motion with respect to an em-

pirical system is known serves the same purpose as would the empirical system itself. This bold innovator might finally refer the words 'true rest' and 'true motion' to his ideal fundamental body and so ascribe true motion and only apparent rest to the earth, thus maintaining a mechanics which would coincide literally with that of ours to-day, except that no small luminous points would be seen sparkling in connection with the fundamental body.

"Hence we see that physical relativism is not a necessary tool of the physicist. Apart, perhaps, from the psychological improbability—of which, however, nothing more positive can be said—the possibility of the development here indicated is logically free from objections throughout, and therefore the same is also true of the possibility of a nonrelativistic physics.

"But I would like to strengthen the argument of Höfler even somewhat further. That is to say, I would ask whether the world in which we live is then really so essentially different from that fictitious one. Imagine the dark roof which conceals the sky placed somewhat higher so that there is room beneath it for the fixed stars, perhaps as the dark background which may be seen nightly in the starry sky. The whole difference then consists in the fact that not only the Foucault pendulum and similar appliances move with reference to the earth, but enormously greater masses as well—all the twinkling lights of the sky by which the thought of a fundamental body in motion with respect to the earth is psychologically greatly facilitated, but logically is not much changed. Now imagine the sky of this earlier dark world suddenly illuminated; then we would see that the fictitious system of reference is closely linked to enormous cosmic masses, and it would be easy enough to accept Mach's hypothesis that these masses condition the fundamental system....

"If a distinction must be drawn between the respective values of the conceptions of Mach and Höfler, it is as follows: Mach's view adds decidedly more to the observed facts; for that it retains physical relativism does not involve freedom from hypothesis, because at best this relativism is theory and not fact. Mach sets up, hypothetically of course, a new formal natural law with regard to the action of masses existing side by side with gravitation, affecting the experiment very materially but unable to raise any claim to the simplest description of actual conditions.

"The other view, which simply introduces the system of reference procured by observation of the terrestrial and celestial movements without asking whence all this is derived, represents the pres-

ent state of our knowledge most adequately without any arbitrary addendum but also without giving the spirit of inquiry any incentive to new experiments.

"It is the old contrast between the most exact and least hypothetical representation possible of the known science, and progressive inquiry after new things in more or less daring and fantastic hypotheses. But Mach in this case stands in the opposite camp as in most other cases where his repugnance to all hypothesis has made him a pioneer in the phenomenological direction....

"I therefore believe I have proved that we can grant the following: Physical phenomena do not depend only on the relative motion of bodies without at the same time admitting the possibility of the concept of an absolute motion in the philosophical sense."*

Strange that Mach, with his reluctance to introduce anything hypothetical except what is absolutely indispensable, should range on the side of the theorists, and after some reflection I believe that there may be a slight hitch in Dr. Frank's interpretation of Mach's view.

First I myself, from my own point of view, would refuse to call the principle of relativity an hypothesis; it is an *a priori* proposition, a theorem, or if you prefer, a postulate of pure thought which either holds good universally, or has no validity whatever. So far as I know, Mach has not discussed this side of the subject but he has instinctively acted upon this view, and I would say that there is a greater hypothetical element in the assumption that the theorem $2 \times 2 = 4$, or any other proposition of the same kind, holds good only for our earth but not for Mars and Venus, than to say that it holds good also for the fixed stars and in the possible worlds outside of our Milky Way. Accordingly, whatever Mach's personal opinion may be, I would regard the universal application of the principle of relativity as less complicated and more free from hypo-

* This last paragraph is printed in spaced letters which indicates the emphasis of the author, and so we print the text of his summary in the original. Dr. Frank says: "Die physikalischen Erscheinungen hängen nicht nur von der Relativbewegung der Körper ab, ohne doch damit die Möglichkeit des Begriffes einer absoluten Bewegung im philosophischen Sinne zuzugeben."

thetical elements than its limitation to a portion of the world.

I can not as yet make up my mind to believe that our system of the Milky Way which furnishes us the grand sight of the fixed stars is an ultimate possessing the characteristics of absolute space.

According to Kant the totality of the fixed stars which are thickest in the Milky Way forms a great system (the system of the Milky Way) and our sun as well as all the visible fixed stars belongs to it. Kant believes that this, our own universe, which in the Milky Way appears to us as an enormous ring but together with the totality of the fixed stars must resemble an oblate spheroid, is not the only cosmic system, but that there are other similar systems outside of it and that they too whirl on through the infinity of space, in company with our Milky Way system, around some center of their own; and this very center of many Milky Ways may partake of a motion the observation of which lies hopelessly beyond our ken. Accordingly the space conditions of the Milky Way may serve *us* as absolute space, but there is a probability that this space is not more absolute than are the space relations in a quick but quietly moving train to the passengers.

Another point where we feel justified in doubting Dr. Frank's exposition is the statement that Mach hypothetically assumes a new law of nature as to the efficacy of masses, besides the law of gravitation. The passage in Mach's writings to which Dr. Frank refers does not (in my opinion) suggest the idea of an additional law of nature according to which the distant fixed stars should exercise a mysterious influence on the Foucault pendulum. We will later on let Mach speak for himself. In our opinion it seems that it would be sufficient to ascribe the rotation of the pendulum to its inertia while the earth revolves round itself and this takes place in the space in which the earth

has its motion, viz., the space of the Milky Way system. The pendulum remains in the plane of oscillation in which it started while the earth turns around underneath. If there are influences at work beyond the expanse of the space of the fixed stars in our Milky Way system, they must affect the totality of our system and would therefore be contained in its space conditions; acting with an unfailing constancy they could not be separated from the properties of our space and would scarcely be discoverable.

There seems to me no need of inventing a new force besides gravitation. The law of inertia seems to explain the Foucault pendulum experiment satisfactorily.

The fixed stars as a totality remain in their places (at least as far as concerns the experiment) and the plane in which the pendulum swings keeps its original direction; thus the apparent motions of both coincide. Their space relations (the space relations of the pendulum and of the fixed stars) are the same, and there is no need to assume the existence of any unknown force exercised by the fixed stars upon the pendulum.

ERNST MACH.

We will let Mach state his views in his own words:

"Obviously it does not matter whether we think of the earth as turning round on its axis, or at rest while the celestial bodies revolve round it. Geometrically these are exactly the same case of a relative rotation of the earth and of the celestial bodies with respect to one another. Only, the first representation is astronomically more convenient and simpler.

"But if we think of the earth at rest and the other celestial bodies revolving round it, there is no flattening of the earth, no Foucault's experiment, and so on—at least according to our usual conception of the law of inertia.

"Now, one can solve the difficulty in two ways: Either all motion is absolute, or our law of inertia is wrongly expressed. Neumann[12] preferred the first supposition, I, the second. The law of

[12] *Ueber die Principien der Galilei-Newton'schen Theorie.* Leipsic, 1870.

inertia must be so conceived that exactly the same thing results from the second supposition as from the first. By this it will be evident that, in its expression, regard must be paid to the masses of the universe.

"In ordinary terrestrial cases, it will answer our purposes quite well to reckon the direction and velocity with respect to the top of a tower or a corner of a room; in ordinary astronomical cases, one or other of the stars will suffice. But because we can also choose other corners of rooms, another pinnacle, or other stars, the view may easily arise that we do not need such a point at all from which to reckon. But this is a mistake; such a system of coordinates has a value only if it can be determined by means of bodies....

"If we wish to apply the law of inertia in an earthquake, the terrestrial points of reference would leave us in the lurch, and, convinced of their uselessness, we would grope after celestial ones. But, with these better ones, the same thing would happen as soon as the stars showed movements which were very noticeable. When the variations of the positions of the fixed stars with respect to one another cannot be disregarded, the laying down of a system of coordinates has reached an end. It ceases to be immaterial whether we take this or that star as point of reference; and we can no longer reduce these systems to one another. We ask for the first time which star we are to choose, and in this case easily see that the stars cannot be treated indifferently, but that because we can give preference to none, the influence of all must be taken into consideration.

"We can, in the application of the law of inertia, disregard any particular body, provided that we have enough other bodies which are fixed with respect to one another. If a tower falls, this does not matter to us; we have others. If Sirius alone, like a shooting star, shot through the heavens, it would not disturb us very much; other stars would be there. But what would become of the law of inertia if the whole of the heavens began to move and the stars swarmed in confusion? How would we apply it then? How would it have to be expressed then? We need not worry about one body as long as we have others enough. Only in the case of a shattering of the universe we learn that all bodies, each with its share, are of importance in the law of inertia....

"Yet another example: A free body, when acted upon by an instantaneous couple, moves so that its central ellipsoid with fixed center rolls without slipping on a tangent-plane parallel to the plane of the couple. This is a motion in consequence of inertia. Here the body

makes very strange motions with respect to the celestial bodies. Now, do we think that these bodies, without which one cannot describe the motion imagined, are without influence on this motion? Does not that to which one must appeal explicitly or implicitly when one wishes to describe a phenomenon belong to the most essential conditions, to the causal nexus of the phenomenon? The distant heavenly bodies have, in our example, no influence on the acceleration, but they have on the velocity."

Now follows the passage to which Dr. Frank obviously refers:

"Now, what share has every mass in the determination of direction and velocity in the law of inertia? No definite answer can be given to this by our experiences. We only know that the share of the nearest masses vanishes in comparison with that of the farthest. We would, then, be able completely to make out the facts known to us if, for example, we were to make the simple supposition that all bodies act in the way of determination proportionately to their masses and independently of the distance, or proportionately to the distance, and so on. Another expression would be: In so far as bodies are so distant from one another that they contribute no noticeable acceleration to one another, all distances vary proportionately to one another."

We do not here understand Mach to fall back on the assumption of a new kind of force, and if we must grant that the distant masses exercise a dominant influence while the influence of the nearest ones (of the earth, the moon, and the sun) vanishes, we would say that this is due to the constancy of the distant masses which, as it were, is an inherent and inalienable part of all mass in the entire system and may be said to characterize its space conditions.

In speaking of "space conditions" I am conscious of using a term which Mach would repudiate, for he claims that for a comprehension of the concatenation of events, the notions of time and space are redundant. He says (*loc. cit.* pp. 60-61):

"To say the least, it is superfluous in our consideration of causality to drag in time and space. Since we only recognize what we

call time and space by certain phenomena, spatial and temporal determinations are only determinations by means of other phenomena. If, for example, we express the positions of earthly bodies as functions of the time, that is to say, as functions of the earth's angle of rotation, we have simply determined the dependence of the positions of the earthly bodies on one another.

"The earth's angle of rotation is very ready to our hand, and thus we easily substitute it for other phenomena which are connected with it but less accessible to us; it is a kind of money which we spend to avoid the inconvenient trading with phenomena, so that the proverb "Time is money" has also here a meaning. We can eliminate time from every law of nature by putting in its place a phenomenon dependent on the earth's angle of rotation.

"The same holds of space. We know positions in space by the affection of our retina, or our optical or other measuring apparatus. And our x, y, z in the equations of physics are, indeed, nothing else than convenient names for these affections. Spatial determinations are, therefore, again determinations of phenomena by means of other phenomena.

"The present tendency of physics is to represent every phenomenon as a function of other phenomena and of certain spatial and temporal positions. If, now, we imagine the spatial and temporal positions replaced in the above manner, in the equations in question, we obtain simply every phenomenon as function of other phenomena.

"Thus the law of causality is sufficiently characterized by saying that it is the presupposition of the mutual dependence of phenomena. Certain idle questions, for example, whether the cause precedes or is simultaneous with the effect, then vanish by themselves."

We understand that Mach endeavors to eliminate the terms time and space, because he wishes to correct the common notion which regards space as a big box into which the world has been packed. Mach says:

"Space and time are not here conceived as independent entities, but as forms of the dependence of the phenomena on one another. I subscribe, then, to the principle of relativity, which is also firmly upheld in my *Mechanics* and *Wärmelehre*."[18]

[18] Cf. "Zeit und Raum physikalisch betrachtet," in *Erkenntnis und Irrtum*, Leipsic, 1905 (2d ed. 1906, pp. 434-448); See also *Space and Geometry*, pp. 94 ff.

We agree with Mach. There is no time in itself; there is no space in itself. Nevertheless, Mach has given much attention to physical space and appreciates the important part which it plays not only in the formation of our space-conception, but also in the actual world, for every spot of space possesses physical qualities according to the particles of mass which are there aggregated. Mach says:

"Since the positions in space of the material parts can be recognized only by their states, we can also say that all the states of the material parts depend upon one another.

"The physical space which I have in mind—and which, at the same time, contains time in itself—is thus nothing other than dependence of phenomena on one another. A complete physics, which would know this fundamental dependence, would have no more need of special considerations of space and time, for these latter considerations would already be included in the former knowledge."

The same idea is expressed by Mach in his Essay "Ueber den Zeitsinn des Ohres:[14]

"Physics sets out to represent every phenomenon as a function of time. The motion of a pendulum serves as the measure of time. Thus, physics really expresses every phenomenon as a function of the length of the pendulum. We may remark that this also happens when forces, say, are represented as functions of the distance; for the conception of force (acceleration) already contains that of time. If one were to succeed in expressing every phenomenon—physical and psychical—as a function of the phenomenon of pendulum-motion, this would only prove that all phenomena are so connected that any one of them can be represented as a function of any other. Physically, then, time is the representability of any phenomenon as a function of any other one."

We do not deny the truth of Mach's view. Nevertheless time and space are very convenient terms denoting two categories of certain interrelations (he would call them interdependencies) in the flux of things. Popular terms mostly originate because there is a need of them,

[14] *Sitzb. der Wien. Akad.*, 1865. Compare *Conservation of Energy*, p. 90.

and it seems to me it would be wiser to correct the errors connected with them than to drop them. If we pursue the latter policy we shall find ourselves obliged to reinvent a new collective term for certain classes of relations which belong together and can not be identified with other relations. The space and time relations are radically different from those of a purely physical, chemical or psychological nature.

We need not fear to retain the old terms, space and time, if we only bear in mind that there is neither absolute space nor absolute time but that the words denote relations. It seems to me that when Kant speaks of the ideality of space and time and insists on their non-existence as objective beings (*Wesen* or *Wesenheiten*) he attempts to say the same as Mach who declares that they are not "independent entities."

The conclusion at which we arrive in considering the nature of time and of space, be it from our standpoint of philosophy or from Mach's physical point of view, may be expressed in one word, that their most obvious characteristic is relativity.

Professor Mach says in one of his notes quoted above, "I subscribe then to the principle of relativity," and so do I. Indeed I go one step further. I consider relativity as an inherent quality of existence and so I adopt the principle of it not as a result of experience but on *a priori* grounds. The principle of relativity, however, is frequently stated by relativity physicists as if the old ideal of science in its objective significance had to be abandoned, as if physics had to be remodeled, and as if the proclamation of the principle of relativity indicated a new departure from our traditional methods. This is not so, and I must insist that the principle of relativity has always been subconsciously in the minds of scientists. Only it has lately

been forced upon the attention of physicists by the progress in astronomical measurements.

How helpful the emphasis recently laid upon the principle of relativity will prove remains to be seen. Its ardent adherents exhibit great zeal which in many directions seems to be misdirected, and it appears to me that in spite of the correctness of the underlying idea their hopes are greatly exaggerated. After a while when the opponents of the principle of relativity will understand that its truth is as much a matter of course as the truth of the law of conservation of matter and energy, the contentions about it will cease and the evolution of science will no longer show evidence of excitement but will continue in its old quiet way.

There is more philosophy in our science than the school of empiricists are inclined to believe. It is very desirable that in familiarizing themselves with philosophy, these scientists should not fall back on the old systems of a visionary absolute, but they should adopt the philosophy of science, the only philosophy which is not a mere ingenious dream, and possesses objective significance.

The philosophy of science is *the* philosophy. It is the indispensable introduction to the study of any science and furnishes the basis for scientific method as well as a general survey of the assured results of all the several sciences. If the philosophy of science had been better known, the principle of relativity had at once been rightly understood and the vagaries of many mystifying contentions would have been avoided.

* * *

In order to appreciate the sense and historic significance of the principle of relativity, we must bear in mind that in dealing with the several innumerable problems of existence science introduces a method which possesses cer-

tain limitations due to conditions which originate through some fictions of an apparently arbitrary nature assumed for the sake of isolating the object of investigation and concentrating upon it our attention.

We must bear in mind that we behold an object by focusing our eyes upon it and that only thereby can we form a picture of the object. It is a fiction to behold an object as if it were a thing by itself and it is positively impossible to see anything as it is in all its relations and with all its changes, past, present and future. Nor would such a comprehension of the object in all its entirety be desirable, for in the omneity of its relations we would see the whole universe while the special feature which concerns us sinks into insignificance. The same is true of science. Each of the several sciences selects its own field of investigation and thus constitutes a definite domain of abstraction for the sake of concentrating all attention upon it. For mechanics and for the measurements of motion in space, we need a reference point which must be able to be considered stationary, and if that is not the case we must refer both the movable place of observation, viz., the reference point (R) and the object observed (O) to one common system, which could be treated as, or must so far as R and O are concerned, actually be, stable.

We repeat that there is nothing absolute; everything is relative; real and actual existences, concrete things and happenings are relative, and if there is any thing that in a certain sense deserves the name absolute it is the truth as described in our mental fictions, the laws of purely formal thought, the eternal uniformities of purely formal relations such as we know from mathematics and all the other purely formal sciences; but even they are absolute only in the sense of constituting an entire system the truth of which is absolute, viz., it stands aloof and is founded in itself as a world of necessary conclusions built up in the field

of anyness to serve as models for any conditions in any world actual or imaginary. And this absolute, this system of mental construction, is after all a system of relations.

The more we ponder on the nature of existence, the more we shall understand the sweeping significance of relativity.

OBJECTIVITY.

Since the dawn of civilization man has groped after truth. He has investigated it; he has pondered on it he has made guesses and proposed hypotheses; he has approximated truth by allegories, foreshadowing it in verse and fable; and since he began to count and to measure he has reduced the results of his inquiry to exact statements

All observations are necessarily subjective, but man is not satisfied with subjective truth, he wants objective truth and objectivity of statement is the ideal of science.

Is objectivity impossible? Must we abandon our idea of science? It seems to us that science has more and more in its various fields approached its ideal of objective truth Standard measures have been invented and perfected. Time is measured by a pendulum of definite size, even apparently trivial factors have been considered such as latitude and altitude; and our precision machines testify to the ingenuity of man's genius in his attempt to eliminate personal equations as much as possible. The reliability of scientific computation has reached a marvelous degree, but it is almost more astonishing that we are still dissatisfied and that our measurements of minute fractions of the wave lengths of light are no longer exact enough for our needs.

In the face of the enormous accomplishments of science in approximating the ideal of objectivity, a new school has risen which goes so far as to deny all objectivity, and in

sisting upon the truth of relativity, it would make us believe that objectivity is a phantom.

The relativity principle was first pronounced by Einstein in the *Jahrbuch der Radioaktivität* (Vol. IV, pp. 411 ff., 1907). It was invented to account for certain difficulties in the explanation of optical and electrical phenomena by considering the relativity of the movements in a system that is not at rest, called a disturbed system in contrast to quiet systems. In all quiet systems the common laws of dynamics hold good and the proposition of the relativity principle has been made for the sake of accounting for the laws of disturbed systems.

The principle of relativity is an *a priori* postulate from which certain theorems are derived whose truth is to be verified or refuted by experiment. Mr. Norman Campbell says:[1]

"The principle is what is more often termed a 'theory'—that is to say, it is a set of propositions from which experimental laws may be logically deduced. It can be proved to be true or false in a manner convincing to everybody only by comparing the laws so deduced with those found experimentally; but a theory which never conflicted with experiment might yet (as I hold) be judged objectionable on other grounds, and, conversely, a theory which was not in complete accord with experiment might yet be judged satisfactory."

Among the postulates of the principle of relativity there is one, counted the second, which presents great difficulties. It proclaims that "The velocity of light determined by all observers who are not accelerated relatively to each other is the same whatever may be the relative velocities of the observer."

An unsophisticated thinker would naturally assume that

[1] See "The Common Sense of Relativity" in *The Philosophical Magazine* for April 1911, pp. 502 ff.

the velocity of light must be expected to increase or decrease according to the velocity of the observer. But the relativist assures us that light is an exception; on his assumption light is like a shadow whose motion depends upon the motion of its body representing the observer. The relation of the shadow to its body remains the same, however its body's (the observer's) velocity may change.

The question as to the velocity of light is a question of physics, not of philosophy, and we will touch upon it later. Here we will state only that the main objection to the relativity principle is the inference which implicates our objective ideal of science.

Not all the relativists agree on all points of their doctrine, and contradictory statements are not uncommon. We can here only characterize the general tendency and will not enter into the individual interpretations too closely.

Relativists try to avoid a difficulty which we grant exists, but is not insurmountable. Idealists of former days have used more subtle methods to dispose of the belief in objectivity of things, of time, and of space. They have produced only quibbles and the relativists have succeeded no better; only it is strange that the movement has originated among the physicists.

In what precedes we have demonstrated the paramount importance of relativity, but for all that we see no necessity for abandoning the old ideal of science. On the contrary we feel inclined to insist on it more strongly than ever. We do not deny the relativity of all existence throughout and without exception, but we still cling to the old scientific ideal of objectivity and we can not see that the relativity principle, in the one-sided sense in which the relativity physicists uphold it, is well established.

Having discussed in the article mentioned the part which relativity plays in scientific method, we feel inclined to add a few suggestions concerning the significance of

the recent movement among physicists who emphasize the principle of relativity and prophesy that through it a new era in the scientific interpretation of the world will have to begin.

We have seen that many of the paradoxes which are proclaimed by the relativity physicists disappear on close inspection, for the contradictions resolve themselves into purely verbal contrasts. The same object is not in itself longer or shorter, but the result of measurement will be different according to the conditions under which the measurements take place. And further, although time can be eliminated, although it may be treated as a function of space, or even be treated as a kind of fourth dimension, the conception of time will nevertheless still remain of great convenience. The truth is that we must subsume time and space under one common category which, with Kant and other thinkers of well-established classical tradition since the days of Aristotle, has been called "form." We must always bear in mind the interrelation between time and space and view the two as the forms of one and the same reality. Time is the form of doing, of progressive action, of change, of events, and space is the form of being, of existence in its juxtaposition of parts. The former is the order of procedure in which the latter is transformed. Neither can be thought without the other, and the two are one. The principle of simplicity requires us to consider both in their interrelation. But for all that the traditional notion of time still proves the best method for rendering measurements of changes intuitively clear while an elimination of time as proposed by the Relativity Physicists is apt to obscure the issue; and we come to the conclusion that experience has not without good reasons found in the proper terms "space" and "time" a very convenient, yea, as it seems to me, the most appropriate, mode of representation.

It is strange that the relativity principle has been proposed for the very purpose of approximating objective truth with greater exactness, but instead of accounting for inexactness or inaccuracies in results and for apparent contradictions by taking into consideration the mistakes in calculation on account of the shifting conditions of this world which is a constant flux, a *panta rhei,* the leaders of the new movement cancel the old ideal of science which has guided us thus far and propose a new standard strongly tinged with subjectivism, built upon the basis of the relativity of all existence.

All experience is a mixture of objectivity and subjectivity: it is due to the interrelation between a sentient subject and the sensed objects. So far science has tried to eliminate the subjective side, the personal equation, while the relativity physicists deny the legitimacy of the ideal of objectivity, or as they call it, the concept of the real. It is true that in clinging to the facts of observation without trying to eliminate the subjective elements and thereby to unify our results in an objective statement, we simplify our calculations, but it is very doubtful whether this procedure can be generally applied to other than optical and electrical phenomena. Relativists deem the theory justified if they simplify their own line of labors. Mr. Campbell exclaims in his enthusiasm:

"Anything more beautifully straightforward it would be hard to conceive. Not only is the result magnificently simple, but it furnishes us with a mathematical instrument of extraordinary power. In place of the elaborate calculations which have hitherto been necessary in dealing with moving systems, all that we have to do now is to solve the problem under consideration for the limiting case of infinitesimal velocity, and then effect a mere algebraical transformation. The only objection that seems likely to be raised is that the principle proves too much, that it appears

impossible that such far-reaching conclusions can be drawn from such simple assumptions: the only difficulty, in fact, is that the thing is too easy."

"The crudest arguments based on the oldest theory of light lead to the conclusion that the rate of a clock as observed by a certain observer must change with the relative motion of clock and observer. For, it will be argued, the observer does not see the clock 'as it really is at the moment,' but 'as it was a time T earlier, where T is the time taken for light to reach the observer.' And on these lines it is easy to show that the apparent rate of a clock moving away from the observer with a velocity v is $(1 - v/c)$ times[3] the rate of the same clocks observed at rest. It is only the magnitude of the change concerning which the two theories differ.

"'Yes,' says our objector, 'that is all very well: of course the apparent rate of the clock changes with motion, but does the real rate change?' We immediately inquire what the 'real rate' means. He is at first inclined to assert that it is the rate observed by an observer traveling with the clock, but when we inquire relative to what clock that observer is to measure the rate he becomes uneasy. He cannot compare another clock traveling with him, for if the 'real rate' of one clock has changed, so has the 'real rate' of the other; and he cannot use a clock which is not traveling with him, because he admits that he does not see such a clock 'as it really is.'

"Pressing our inquiries, I think we shall get an answer of this nature. 'If I take a pendulum clock to some place where gravity is different, the rate of the clock will change. It is a change of this nature which I call a change in the "real rate," and I want to know whether there is any change of that kind, on the theory of relativity, when the

[3] c denotes the universal velocity whatever it may turn out to be. See *ibid.* p. 508.

clock is set in motion.' Now why does our objector call a change of the first kind a change in the 'real rate'? The reply is to be found in the history of the word 'real.' The word is intimately associated with the philosophic doctrine of realism, which holds that the most important thing that we can know about any body is not what we observe about it, but its 'real nature,' which is something that is independent of observation.

Now, of course, a quantity which is wholly independent of observation cannot play any part in an experimental science, but there are quantities which are independent of observation in the more limited sense that they are observed to be the same by whatever observer the observation is made. The term 'real' has come to be transferred from the philosophical conception to such quantities. The 'real rate' of the clock is said to change when it is transferred to a place where gravitation is different, because all observers agree that the rate of the clock which has been moved has undergone an alteration relatively to that which has not been moved.

"Now in the conditions which we are considering the observers do not agree. If A and B, each carrying a clock with him, are moving relatively to each other, they will not agree as to the rate of either of their clocks relative to A's standard or to B's standard or to any other standard. The conditions which, in the case of the alteration of gravitation, gave rise to the conception of a 'real rate' are not present: in this case there is no 'real rate,' and it is as absurd to ask whether it has changed as it would be to ask a question about the properties of a round square. However, some people, who in their eagerness to escape the reproach of being metaphysicians have adopted without inquiry the oldest and least satisfactory metaphysical doctrines, are so enamoured of the conception of 'reality' that they refuse to give it up. Finding that the observations of different ob-

servers do not agree, they define a new function of those observations, such that it is the same for all observers, and proceed to call this the 'real rate.' This function, according to the principle of relativity, is $\beta n'$ where n' is the rate of the clock as seen by an observer relative to whom it is traveling with the velocity v: according to that principle, if we substitute in that function the appropriate values for any one observer, the resulting number will always be the same. So far no overwhelming objection can be raised."

What the relativists call "real" we would call objective, and we deem the ideal of objectivity to be the goal of science. Mr. Campbell has much to say on the concept of reality:

"It is the great merit of the principle of relativity that it forces on our attention the true nature of the concepts of 'real time' and 'real space' which have caused such endless confusion. If we mean by them quantities which are directly observed to be the same by all observers, there simply is no real space and real time. If we mean by them, as apparently we do mean nowadays, functions of the directly observed quantities which are the same for all observers, then they are derivative conceptions which depend for their meaning on the acceptance of some theory as to how the directly observed quantities will vary with the motion, position, etc. of the observers. 'Real' quantities can never be the starting point of a scientific argument; by their very nature they are not quantities which can be determined by a single observation: the term 'real' has always kept its original meaning of some property of a body which is not observed simply.

"All the difficulties and apparent paradoxes of the principle of relativity will vanish if the attention is kept rigidly fixed upon the quantities which are actually observed. If any one thinks he discovers that that principle predicts some experimental result which is incomprehensible, let

him dismiss utterly from his mind the conception of reality. Let him imagine himself in the laboratory actually performing the experiment: let him consider the numbers which he will record in his note-book and the subsequent calculation which he will make. He may then find that the result is somewhat unexpected—to meet with unexpected results is the usual end of performing experiments,—but he will not find any contradiction or any conclusion which is not quite as simple as that which he expected.

"There is one further point sometimes raised in connection with the principle on which a few words may be said.

"It is sometimes objected that the principle 'has no physical meaning,' that it destroys utterly the old theory of light based on an elastic ether and puts nothing in its place, that, in fact, it sacrifices the needs of the physical to the needs of the mathematical instinct. That the statement is true there can be no doubt, but the absence of any substitute for the elastic ether theory of light may simply be due to the fact that the principle has been developed so far chiefly by people who are primarily mathematicians. It is well to ask, can any physical theory of light be produced which is consistent with the principle?

"The answer depends on what is meant by a 'physical theory.' Hitherto the term has always meant a 'mechanical theory,' a theory of which the fundamental propositions are statements about particles moving according to the Newtonian dynamical formulæ. In this sense a physical theory is impossible if the principle of relativity be accepted, for the same reason that a corpuscular theory of light is impossible, if the undulatory theory of light be accepted. Newtonian dynamics and the principle of relativity are two theories which deal in part with the same range of facts; they both pretend to be able to predict how the properties of observed systems will be altered by movement. If they

are not logically equivalent they must be contradictory: in either case an 'explanation' of one in terms of the other is impossible. It can be easily shown that they are contradictory: if the principle of relativity is true, Newtonian dynamics must be abandoned."[4]

We start with "the facts of observation," and try to establish the objective state of things, called also "the real"; but relativists ignore the latter, and since every observer has his own particular observation, they declare that there is neither real time nor real space. The real is ruled out from observation.

Suppose, however, that the clocks which the relativist observes were the heartbeats of the relativist himself and the observer were the diagnosing physician, would the relativist insist that the physician had better drop out of sight the notion of reality, that there is as little sense in asking for "the real rate" of his heartbeat as it is absurd "to inquire whether, if all triangles had four sides, all circles would be square"?[5] If we can not attain an absolutely correct objective statement, we keep at least the ideal in view and this ideal is not an empty dream.

The relativity principle is a mathematical view of certain problems worked out for the sake of most minute measurements; and the attitude of the relativists is stern. If the facts can not be clearly represented by it, the worse for the facts, and if the physicists declare that their physical theories are incompatible with it, a new brand of physicists has to be manufactured who will inaugurate a relativist reform in physics.

PRIMARY CONCEPTS.

The relativity problem would never have originated had the philosophy of science been clearly and distinctly

[4] This conclusion is reached by Sommerfeld in a paper, *Ann. d. Phys.*, XXXIII, p. 684, etc. (1910).

[5] See Campbell, *loc, cit.*, p. 509. The comparison is not appropriate.

understood by physicists, but they have familiarized themselves very little with even the problems, let alone reached proper solutions which explain the elementary concepts of our scientific terms, the difference between substance and form, between energy and matter, and the significance of the purely formal sciences.

As mathematicians are in the habit of starting with axioms, so the relativists begin with postulates and these postulates come in collision with the primary concepts such as have been formulated among the orthodox physicists and mathematicians of the present day.

A truly scientific view will brook neither axioms in mathematics, nor postulates in philosophy, nor primary concepts in physics.

There has been much talk about primary concepts, and arguments have been offered why time is not a primary notion or why we should let it pass as such. The truth is that time as well as space are two methods of describing definite relations. Time is not so much a fourth dimension of space, though we might look upon it as if it were such, time is the measure of motion and space is the scope of motion. Both time and space are presupposed in the idea of motion. There is no time in itself, there is no space in itself. What Newton and others with him call absolute space is "space conception" and what they call absolute time is "time conception." Such are the ideas which by pure deduction on *a priori* arguments, physicists form of time and of space, just as mathematicians formulate the general conception of numbers, of distances and of other relations, angles, areas, etc.

The idea of primary concepts is a very unfortunate device to lay a foundation for science. The faults of this method will not show so long as specialists are concerned about specialist problems, but the carelessness of taking anything for granted shows itself as soon as any problem

broadens out into a general inquiry when its connection with universal problems is questioned. Such primary concepts are assumed to be undefinable and self-evident. That opens the door to an arbitrary interpretation as to the nature of space and time and energy, and gives a wide berth to mysticism.

Science brooks neither axioms nor primary concepts. Science starts with experience; it quarries out of experience the stones of the purely formal sciences which furnish all the methods of both common sense knowledge and scientific inquiry. The most general characteristic of experience is activity. Activity manifests itself in change. Change implies motion; it means either change of place, i. e., moving from here to there, or change of combination, viz., a moving of particles among themselves. Change interferes with existing relations, it modifies the old interrelations and establishes new interrelations.

The nature of relations in one terse term is called form. The word "form" comprises both outer shape and inner structure, and all interrelations of things as well as thoughts can be determined by the laws of pure form, arithmetic, geometry, logic, etc. Under all circumstances change modifies relations and means "transformation." There is a transformation in the juxtaposition of things or their parts, and there is a succession of events. The scope of the former we call "space," of the latter "time"; or better from the former we deduce our notion of space, from the latter our notion of time.

Physical inquiry is not helped by calling certain features of experience "primary concepts" and least of all (as has been done) should space, time and force,—these highly complicated constructions of *a priori* thought— be beclouded by this mystifying name. Both time and space are features of the form of existence, and force is a general term for that feature of existence which marks its activity

as motion, viz., as change of place, or rather as that which causes changes and is measured by the resistance overcome.

If we adopt the relativist principle to ignore the scientific ideal of objectivity, i. e., if we define size as the result of measurement and moments of time as determinations of measurement by units of duration, without regard to the ideal of coincidental happenings, and a common standard of time, we may produce incredible statements against which common sense rebels, and Professor Magie in his Presidential Address,[6] delivered before the Physical Society and Section B of the American Association for the Advancement of Science, at Washington, D. C. (December 28, 1911), says in comment thereof:

"A description of phenomena in terms of four dimensions in space would be unsatisfactory to me as an explanation, because by no stretch of my imagination can I make myself believe in the reality of a fourth dimension. The description of phenomena in terms of a time which is a function of the velocity of the body on which I reside will be, I fear, equally unsatisfactory to me, because, try I ever so hard, I can not make myself realize that such a time is conceivable......I do not believe that there is any man now living who can assert with truth that he can conceive a time which is a function of velocity or is willing to go to the stake for the conviction that his 'now' is another man's 'future' or still another man's 'past.'

"One of the members of this society, recognizing our present inability to conceive of relative time, and conceiving our intuitions of space and time to be the result of heredity operating through many generations of men who lacked the light of relativity, once proposed to me that every one who could get even a glimmer of the notion of relative time should persistently exercise his mind therein

[6] Published in *Science*, February 23, 1912, pp. 281 ff:

and teach it to his students, in the hope that in a few generations the notion would emerge with the force of an intuition. It would not be fair to leave the impression that he was solemnly serious when he made this suggestion."

Form (i. e., relativity) is, as much as matter and energy, an ultimate generalization and may be called a fundamental concept (not a primary concept), and all the work of science is a tracing of transformations.

It is essential for the measurement of space and time to employ as measures uniform units, for space of distance and for time of duration. In the same way we need uniform units to measure force.

Besides a quantitative analysis of experience, there is a qualitative analysis which traces such transformations as build up parts into a higher unit, whereby through the interrelation or the interaction of the parts a new thing originates possessed of properties which are absent in the parts before their combination.[7]

The law of change is called causality. Cause is the motion which starts the process of transformation; effect is the result of the change; and reason is the general rule (formulated as a so-called law of nature) from which we understand why the cause must have this effect.[8]

The so-called law of the conservation of matter and energy is a deduction from the law of causality, which can be made as soon as we understand that all happenings are transformations, for if all changes are transformation, the amount of existence, its *that*, remains the same, only its form changes.

While investigating the several problems of our experience, scientists assume that they deal with real occurrences and thus they implicitly grant the *that* of existence,

[7] See for instance the author's exposition of the nature of quality in *The Monist*, Vol. XV, p. 375. See also *Philosophy of Form*, p. 12.

[8] This has been repeatedly discussed, e. g., in the author's *Fundamental Problems*, pp. 79 ff.

popularly denoted "matter" and "energy," viz., thingishness (or with a Latin term "reality") and actuality. The existence of ether is but an extension of the concept matter and so physicists have so far believed in the existence of ether; but the relativity physicists, in their anxiety to propound original ideas, deny the existence of ether. Says Prof. William Francis Magie in his above mentioned Presidential Address:

"The principle of relativity in this metaphysical form professes to be able to abandon the hypothesis of an ether. All the necessary descriptions of the crucial experiments in optics and electricity by which the theories of the universe are now being tested can be given without the use of that hypothesis. Indeed the principle asserts our inability even to determine any one frame or reference that can be distinguished from another, or, what means the same thing, to detect any relative motion of the earth and the ether, and so to ascribe to the ether any sort of motion; from which it is concluded that the philosophical course is to abandon the concept of the ether altogether. I may venture to say that in my opinion the abandonment of the hypothesis of an ether at the present time is a great and serious retrograde step in the development of speculative physics. The principle of relativity accounts for the negative result of the experiment of Michelson and Morley, but without an ether how do we account for the interference phenomena which made that experiment possible? There are only two ways yet thought of to account for the passage of light through space. Are the supporters of the theory of relativity going to return to the corpuscles of Newton? ......There is choice only between corpuscles and a medium, and I submit that it is incumbent upon the advocates of the new views to propose and develop an explanation of the transmission of light and of the phenomena which

have been interpreted for so long as demonstrating its periodicity. Otherwise they are asking us to abandon what has furnished a sound basis for the interpretation of phenomena and for constructive work in order to preserve the universality of a metaphysical postulate."

The concepts substance, i. e., matter or mass, and energy are ultimate generalizations as much as form, but they are very different from form. We could do without the words "matter" or "ether" by the use of some other indication to be introduced in our formulas which denotes reality; but that would not disprove the truth of the popular view, which describes every concrete bodily existence as material, nor is it likely that the old method of nomenclature will be rendered antiquated or erroneous.

We must not forget what matter means. Matter is a word which denotes that quality of objects which all of them have in common, viz., objectivity. An object is a thing that is objected to us, that offers us resistance, that impresses itself upon our existence and thereby affects our senses, and by objectivity we understand the general property of concrete existence, the *that* of experience, or its reality, viz., its thingishness. To deny the reality of the real, the thingishness of things, is as ridiculous as the opposite mistake, i. e., to think of reality, or objectivity, or of matter as a mysterious entity in itself. There is no reality *in abstracto*, for every *that* of existence is of a definite form which acts somehow, and the activity of things we call their actuality, or, as we call it in physics, energy.

The same problem presents itself in the domain of the phenomena of ether, i. e., of light and electricity. There are some good reasons to assume that concrete matter has originated by a contraction or condensation of a more primitive substance which for all we know may prove to be the luminiferous ether, that thin substance which has been assumed to be the medium of light and electricity. If it is

claimed by modern physicists that the principle of relativity disposes of the ether, that we no longer need it and can discard a belief in it as a superstition, that all physical phenomena can be accounted for without the assumption of an ether, we confront the same situation as in the theory of energetics, where the claim is made "There is no matter. all is energy."

The truth of this position, so far as we freely grant it, is this, that all scientific explanation describes the transformation of things; it traces the changes that take place according to the laws of form (mathematics and mechanics). In experience we are confronted with the fact that it is so, but the scientist inquires into the factors how it has become so, *how* it acts, and *how* it changes. By describing the *how* in formulas (so-called laws of nature) we denote the several factors with algebraic letters, such as $g =$ gravity, $t =$ seconds of time, $d =$ the distance traversed by a falling body and $v =$ the velocity of the fall, etc., and express their interrelation in equations, as

$$v = gt \text{ and } d = \tfrac{1}{2}gt^2.$$

By this method the essential features of natural phenomena are expressed in symbols, and he who has been initiated into the secret meaning of the symbols and the method of using them, will be able to predict the course of events if he is in possession of the necessary data.

What we here call with one word "essential" Kirchhoff characterizes in two words "most complete and most terse," or to use the common version "the most exhaustive and simplest." We deem our term preferable, and we understand by "essential" all that which is efficient to produce the result, not more, not less.

We speak of the three laws of Kepler and of the condensed statements of the law of gravitation as "formulas," and this term truly expresses the nature of these general-

ized descriptions of certain types of uniformities. They are reductions of events to their purely formal (i. e., purely relational) conditions, and these purely formal conditions are the determinant (i. e., the causative) factors in all possible phenomena of a special type.

This is not a new truth. How old it is may be inferred from the Greek term "formal"[9] which in its etymology means "the causal" or "the causative" because the Greek philosophers describe the formal factors as efficient in causation.

When we have traced the essential factors of a certain type of changes, the scientist's work is finished. Whether mankind will ever be able to complete a scientific comprehension of the world in all its details, must be regarded as doubtful, but wherever science has succeeded in discovering the essential factors and has reduced them to formulas, we have been enabled to offer for every such phenomenon a satisfactory explanation.

This procedure affords us an insight into the reason why the course of a certain phenomenon must be so, why it can not be otherwise, and in this procedure the *that* is the basis, the *how* is the method of cognition. There is no explanation possible for the *that*, for the reality of the real, for the actuality of the fact; all explanations refer to the *how*. The *that* is a brutal fact, and the ultimate goal of science is the how, the answer being the formulation of laws of nature which explain to us by a use of the law of pure form that under given circumstances definite transformations will take place. Knowledge of the laws of nature helps man to adapt himself to nature and also to adjust his surrounding natural conditions to himself.

In our explanation we can omit the *that* as a matter of course, for it is understood that reality is real. We can describe the purely formal relations only, which are the

[9] *τὸ αἰτιῶδες*, derived from *αἰτία* = cause.

essential part of explanations. There is no sense in explaining the *that*. We have simply to state whether or not a formula covers actual facts, but to deny the *that* and say that there is only a how the world wags, but there is no world, seems to us a proposition that misconceives the situation.

We must not forget that such a word as substance, denoting here both "matter" and "ether" or existence in general, is a term that stands for objective reality. Ether is the *that* of the phenomena of electricity and light, as matter is the *that* of bodily objects, declaring that they are real, that they are concrete, and the term "substance" covers any kind of existence, it embraces both matter and ether or whatever the ultimate world-stuff may be called. There is no sense in denying their actuality, and all that may be meant by such a denial can only be either the redundancy of an express declaration that the formulas of physics refer to real processes, or a denial of ether or of matter as existences in themselves apart from their manifestations in definite configurations or modes of motion — a proposition which nowadays no one will seriously dispute.

A denial of the existence of substance (of matter and ether) is a purely verbal quibble. We might as well deny the existence of energy and declare that there is no energy, that there are only changes of place. The truth is that the faculty of existence which manifests itself in changes of place is called energy. We must not conceive of energy as something in itself.

* * *

I am told that my own view is the gist of the principle of relativity, and if that be true, I would gladly hail a philosophy of relativity as another name for the philosophy of science. I have myself characterized the philosophy of science as a philosophy of form, and form denotes the relations in their totality. However, I would add that the

system in which I have formulated this philosophy of science is simpler than the world-conception of the relativity physicists, besides it rests on a more solid foundation and is absolutely free from paradoxes.

While I deny that we can dispense with the idea of objectivity (be it called matter, or ether, or substance) I claim that we need make no mention of it in our formulas. In this sense we can dispense with the mention of ether. While I would not take the several paradoxes of time and space as serious and deny their objective truth, I grant that by a little confusion of thought in calling time or space relations the results of our different measurements, we can legitimately produce these paradoxes by exhibiting the inevitable discrepancies which originate through measurements from different standpoints as objective contradictions. Finally I consider it the ideal of a scientific philosophy to reduce all possible occurrences to relations, to resolve them into questions of form, to look upon them as transformations, and therefore I say that the ultimate aim of science is to describe everything in formulas. I see no objection to the relativist claim that this is a postulate of science. In fact, I deduce this postulate directly from my conception of reality which presents itself everywhere in our experience as transformation. Thus we would justify the principle of relativity on the basis of the old traditional basis of exact science.

The main claim of the relativists is based upon their simplification of the electromagnetic equations, and this is granted even by the adversaries of the principle of relativity. Professor Magie says:

"It is surely true that if it were not for this demand of simplicity, immediately attainable and at present expressed in the electromagnetic equations, the chief incentive to the development of the theory of relativity would be wanting."

The one simplification of formulas is attempted by certain relativists by a generalization of time and space into a higher four-dimensional system, and they call it a four-dimensional space. We may note incidentally that Wagner's Parsival has anticipated the doctrine of relativity, for in his search he utters the mysterious words: "*Zum Raum wird hier die Zeit!*" (Into space here changeth time!) The relativists might as well have called their four-dimensional space a four-dimensional time. We abstain from giving it a name, but subsume time and space under one and the same category as "form" which enables us to view time and space as two inseparable factors of the cosmic system of interrelations, and we deem it wise to remember that they are different. If the relativity physicists have this in mind and do not mean ulterior mystifications, I would not hesitate to join their ranks on this point.

* * *

We may add one more comment about simplification. Logical possibilities are wider than actualized reality. Reality is one instance among many others which are not actualized. The fictions of fairy tales, of Gulliver's Travels, and of religious myth are instances of it. But in the domain of pure logic even actually absurd conditions parade as legitimate potentialities. Actual space has three dimensions, but metageometricians have invented more-dimensional spaces. Why not? We have in the construction of purely logical systems the undeniable right to generalize into the not actualized logical possibilities and mathematicians can not be restrained from building up a pangeometry. While Euclidean space is homaloidal, they may create all kinds of curved spaces, which are all legitimate before the tribunal of pure logic, if they are but consistent in themselves. The main gain derived from such constructions which will naturally appear to the average man of average common sense as gratuitous, if not positively non-

sensical, consists in rising to a higher level and understanding from this higher point of view the actualized reality better than if he remains on the *terra firma* of a limited sense-experience.

It might help our comprehension of causality as a transformation according to the laws of form to conceive the chain of causation as reversible, that the condition of causes are turned into effects and that the final factors that bring about the effect become the causes. This view has been humorously worked out by Fechner who for this purpose assumes that the pendulum of events will go on for a while in the direction it takes now, but the time will come when it will swing back. And then it will appear to us as quite natural and necessary that the decayed and waste material from fields and polluted rivers pass into our bodies and are changed in our bowels into juice to go forth from our mouths on the dinner table as lovely fruit or cheese, with bread and butter, and as roast venison or fish to go back and constitute useful parts in the revived animal. It would please us to see all this come about and the thought of the resurrection of the lamb under the butcher's knife would demonstrate that there is a purpose in the law of causation. We would be accustomed to the outcome and deem it natural. In fact some notions of an inverse world order in the golden age when the lamb will feed on the wolf, when the deer will hunt the hunter, when the rich shall be poor and the poor rich, when the miserable will be comforted while the fortunate will be tortured has now and then received serious support in the religious hopes of the disinherited classes of mankind, and we may find in the New Testament an echo of this belief in those traditions which come down to us from Ebionite sources, the parables of the foolishness of the rich and the benediction of the poor. Dives goes to Hell while Lazarus is carried by angels to Abraham's bosom. Abraham says in Luke xvi: 25: "Son,

remember that thou in thy lifetime receivedst thy good things, and likewise Lazarus evil things: but now he is comforted, and thou art tormented." No mention is made that Dives was wicked and that Lazarus was good; the only argument is that the other world must be reverted in its order.

A view of this kind which generalizes the mechanical constitution of the world and sees the possibility of an inverted causation, just as an engine may be reversed, may widen our comprehension and simplify our formulas of moral action, but we need not for that reason believe in its actualization. It is simply an instructive *lusus imaginationis,* an ingenious and helpful fiction—like our conception of four-dimensional space.

The mathematician who limits his studies to the Euclidean plane will understand his problems better if he becomes familiar with the theorems of stereometry, or if he views the figures of plane geometry as projections; or again if he regards a certain set of curves as conic sections. And further many problems of stereometry find a simpler formulation if viewed from the more comprehensive, though purely imaginary, view-point of a four-dimensional geometry. All this indicates that the simplifications of which the relativity physicists boast, may be (and I am inclined to believe that they are) very harmless. For all I can say, judging merely from the acceptance they have found, they must be true, but I can not see why they should be subversive of the scientific world-conception of the past.

A peculiar view of time which has been proposed in all seriousness, although common sense might consider it as absurd, is the concept of time and space as consisting of discrete ultimate units. Do not our years, and days, and our hours too begin at definite moments? We become fifty or sixty years old suddenly with the beginning of a definite minute, According to this, time would run in jerks like

the jumping second hands, and it would ultimately consist of infinitesimally small units of duration. Space also would be stippled and not continuous. Every motion would have to proceed in hopping from spot to spot, and the surface of a plane would be not unlike a half-tone picture which produces the impression of a continuous level but consists in reality of different dots more or less deeply tinged with ink. Such conceptions of time and space are quite conceivable although our classical and well-established views of both present them as continua. If space and time were actual entities endowed with positive qualities, if they were not merely potentialities of motion, a scope in which we move about, we could discover the nature of space by experiment. However, as they are constructions made in the abstract domain of anyness we should not refuse to consider seriously all kinds of propositions as to the nature of time and space.[10]

In comment on theories of this kind we would say that duration is continuous, but time consists of discrete units of duration; and again the scope of motion shows us an uninterrupted expanse while geometry exhibits definite lines of definite direction and of definite length. Geometrical space in its classical Euclidean form is not stippled, nevertheless every construction is particular. Geometrical points have no extension, but they possess a definite location, being determined, e. g., by two crossing lines. Thus space is

[10] *The Monist* of October 1912 contained an article on "Atomic Theories of Energy" by Mr. Arthur E. Bostwick, which is of interest both to those who accept and those who do not accept this theory. In comment we would say that Mr. Bostwick's defense of an atomic theory of energy is certainly true of definite amounts of energy, and his theory holds good also in his comparison of energy to amounts of money values deposited in a bank account. If deposits were made in specie, we could trace every dollar of a deposit. It is true we can not do so, but this we can not do only because no one cares to receive definite and individual coins, but is satisfied with money in any form. Therefore the bank is like a reservoir of water which receives and gives out water as it happens to come. The bank gives credit for amounts received and pays out amounts according to request. Thus the individual coin is lost sight of as the many drops of water are definite and concrete masses, and every dollar in a bank represents some concrete value somewhere.

not the totality of all points, but the totality of our scope of motion and anywhere in space points may be laid down. In a word: Time and geometrical space are constructions invented for the purpose of making measurements possible in a scope of potentialities.

Actual existence is always definite, pure forms however as well as purely formal thoughts, are always potential.

It seems as if the beginning of actuality must consist in establishing something that is limited and concrete. In this way it appears plausible that a potential world would be continuous as an ocean of pure ether might be, but an actual world ought to consist of a group of units, of atoms, of definite particular specks of existence endowed with definite amounts of energy, and we ought to be able to trace every definite amount of existence through all the changes which in the process of evolution it will undergo; and this ought to be true as regards every amount of both matter and energy.

SOME PHYSICAL PROBLEMS OF RELATIVITY.

The physical problems presenting themselves in the experiments which have become connected with the movement of relativity do not seem to have any direct bearing on the principle of relativity itself. Relations are of a purely formal nature and relativity therefore belongs to the same kind of knowledge as arithmetic, geometry and logic. Relativity can and must be applied to physics just as much as there is an applied mathematics, but as the Pythagorean theorem is independent from its applications in experience, so applied relativity can neither establish nor refute the principle of relativity. This is true above all of the well-known and most important Michelson-Morley experiment.

The instrument made in Berlin by Schmidt & Haensch was so delicate that it was of no use in Berlin, and even

when placed upon the foundation for the pier of the equatorial in the Astrophysical Observatory at Potsdam the fringe of interference rings disappeared by stamping upon the pavement at a distance of about 100 meters. Every detail of consequence was taken into consideration, not only the motion of the earth through the ether but also the motion of the whole solar system towards the constellation of Heracles. The expansion of the brass arms of the instrument through a change in temperature, and also the bending of the arms through rotation were duly considered and the difficulties arising therefrom met. A scale ruled on glass was employed in order to dispense with the micrometer screw which here proved useless. Yellow light was used, because its wavelength is least difficult to measure.

If the ether is at rest while the earth moves through it, the time required for light to pass from one point to another on the earth's surface would depend on the direction in which it travels. Two pencils of light that travel over paths at right angles to each other will interfere; the one traveling in the direction of the earth's motion will travel 0.04 of a wave length farther than it would have done were the earth at rest, while the other pencil at right angles to the motion of the earth would not be affected.

The results of Professor Michelson's experiment are negative. He found very small displacements in the fringes of his ray of light, so small that they must be accounted as mere errors of the experiment. While we ought to expect a displacement of 0.05 we have only such as lie between 0.004 and 0.015. Professor Michelson says:[11]

"The interpretation of these results is that there is no displacement of the interference bands. The result of the hypothesis of a stationary ether is thus shown to be in-

[11] "The Relative Motion of the Earth and the Luminiferous Ether" in *The American Journal of Science*, Vol. CXXII, page 128.

correct, and the necessary conclusion follows that the hypothesis is erroneous.

"This conclusion directly contradicts the explanation of the phenomenon of aberration which has been hitherto generally accepted, and which presupposes that the earth moves through the ether, the latter remaining at rest."

In another article Professor Michelson states his result thus:[12]

"The luminiferous ether is entirely unaffected by the motion of the matter which it permeates."

Professor Michelson has varied the conditions of his experiment by trying whether deviations could be detected through a change of level, by throwing pencils of light upward and by repeating it at different hours of the day, but the displacements remained insignificant. One of Professor Michelson's articles ends thus:[13]

"In any case we are driven to extraordinary conclusions, and the choice lies between these three:

"1. The earth passes through the ether (or rather allows the ether to pass through its entire mass) without appreciable influence.

"2. The length of all bodies is altered (equally?) by their motion through the ether.[14]

"3. The earth in its motion drags with it the ether even at distances of many thousand kilometers from its surface."

Another article by Professor Michelson on the same subject is published in *The American Journal of Science*, Vol. CXXXIV, p. 333.

What this famous experiment has to do with the principle of relativity except in a most general way, is not yet clear to those who have not joined the ranks of the rela-

[12] "Influence of Motion of the Medium on the Velocity of Light," in *The American Journal of Science*, Vol. CXXXI, page 386.

[13] "The Relative Motion of the Earth and the Ether," *The American Journal of Science*, Vol. CLIII, p. 478.

[14] This would be the case according to the theory of H. A. Lorentz, whose views are mainly presented in the *Encyclopädie der math. Wissenschaften.*

tivity physicists; but the relativity physicists insist very vigorously and dogmatically that it proves, or at least favors, their theory. Professor Michelson himself has not joined their ranks, though he recognizes the difficulties of the situation.

It is strange that Michelson's experiment seems to stand in contradiction to another and older experiment made first by Bradley, which is known as the aberration of light. If the earth passes through the ether with its own velocity (e) while the rays of the sun come down upon the earth with the velocity of light (l) there ought to be a deflection of light amounting to e/l, viz., the velocity of the earth divided by the velocity of the light in its path from the sun towards the earth, and though this relation is very small, it has actually been observed and determined to amount to a trifle over twenty seconds.

This conclusion which could be anticipated according to the logic of mechanics seems to be contradicted by Michelson-Morley's experiment in which the attempt is made to measure with a ray of light the motion of the earth while passing through the ether.

The discrepancy between the two experiments will perhaps find a proper explanation in the proposition that if the source of light lies outside the earth as in the case of the rays of the sun, they will show the deflection. As is to be expected they would come down in straight lines like raindrops falling in an absolutely quiet air which would be caught by a moving body as if they came down at an angle; but if the source of light moves along with the earth there would be no difference whichever way they turn, first towards the east or first towards the west, or at right angles, and the sources of the light would partake of the acceleration of the earth so as to show no difference, as raindrops dripping down within the car would fall down in straight lines from its top to the floor, assuming that the

doors and windows of the car are hermetically closed and there be no draft which would deflect their perpendicular dripping.

It almost seems as if some ether were carried along by the earth to a considerable distance beyond its surface while the other ether in outer space would remain at rest, but it would be bold for any one but a specialist to venture the proposition of any theory on so new a subject of which few facts only have been ascertained. Yet most assuredly the topic under investigation has nothing to do with the principle of relativity, unless relativity is a misnomer for the phenomena attributed to the luminiferous ether.

The question of relativity is a philosophical problem, but the Michelson-Morley experiment is of a purely physical nature, and so we must expect that the last word as to its explanation should be given by physicists.

The other experiment which is assumed to verify the principle of relativity is the one first made by Kauffmann, and afterwards repeated in a modified form by Bucherer. This experiment too has little or nothing to do with relativity. On the contrary it seems to prove the existence of something absolute for it reaches a limit of velocity.

There is at present a tendency in the world of thought, noticeable in pragmatism and other anti-intellectual movements, which seems to annihilate the very existence of objectivity, and with it science, man's endeavor after a purely objective cognition. Everything is relative, and the general belief has spread that an absolutely objective description is impossible. To speak of the size of objects seems to have lost its sense, for size has become to the present generation merely the result of measurement, and thus an objective determination is in some quarters looked upon as a superstition of prescientific tradition, an inheritance from the dark ages. But it is not true that there is

no objectivity, for one of the greatest accomplishments of Michelson was the establishment of a definite measure by calculating the size of a meter in wave-lengths or red cadmium light in a vacuum. The waves of light are absolutely definite, and thus we have here a result of measurement in truly objective terms. If the Kauffmann-Bucherer experiments prove, as is claimed, that an increase of velocity means an increase of mass and that the limit which is reached is the velocity of light, we only learn that relativity is not without bounds, and that on the contrary a climax is reached which can not be surpassed. The highest velocity is the velocity of light.

The conclusion that the highest velocity is the velocity of light seems to be contradicted by the facts of gravitation for according to the Newtonian theory gravitation is possessed of a practically infinite velocity in that the gravity of the sun exercises its influence upon the planets without any perceptible difference of time. But this is no objection, for consider: The action of gravity formulated in the well-known law of falling bodies and of their acceleration which describes true motions is very slow in comparison to the velocity of light. The influence which is exercised in the strain between two gravitating bodies, say between the moon and the earth, is not a motion at all, but a condition, and this condition is the same between the two centers of the thus interrelated bodies. It is a state of tension and there is no transference of a wave motion either from the moon to the earth or from the earth to the moon. The tension is simultaneous. The misconception seems to rise from the error that there are two bodies and there is a third item which manifests itself as a passing from the one to the other under the name of gravitation. We must view the whole system as one field of action in which several bodies in motion are balanced among themselves according to their mass. Their mutual attraction is not

transferred motion but a simultaneous interaction. Newton retarded the general acceptance of the law of gravitation, first definitely proposed by Hooke, for eighteen long years because he could not make up his mind to believe in an *actio in distans,* and when he was finally convinced, he still expressed his misgivings how to overcome this objection, but is there any *actio in distans* at all? Is not the whole system of the universe an interrelated whole and does not a center of gravity (howsoever it may have originated) extend so far as its stress reaches? Where its strain produces a tension, there it affects its surroundings. If we look upon the phenomena of gravitation in this light we need not make the fantastical assumption that gravity is possessed of an infinite velocity.

The relation between the increase of velocity and the increase of mass promises to throw light on the ultimate constitution of matter, but the result of the experiment is only the first step to a solution of this tremendous problem, concerning which at the present stage of science we can have only vague suggestions. When the man appears who can read the facts aright, he may be able to point out how by a mere stress the aboriginal world-stuff which, for all we know, may be, or even must be, the ether, produces a tension within this mysterious infinitely elastic and incredibly thin substance, and the tension between two centers of such contraction would, like the strain between nodes within thin tridimensional rubber, act in all directions according to the Newtonian formula of gravitation, as being directly proportional to the product of their amounts of contraction, and inversely proportional to the square of their distance between two centers. Thus the origin of matter would be due to an unknown force which with a velocity only inferior to the velocity of light would drive infinitely small corpuscles around in a whirling dance with

such a regulated speed that conglomerated multitudes of such whirls would appear to us as solid masses.

Here again we would be confronted by an ultimate limit. We would discover that objective reality, our world of matter in motion, is built up of ultimate particles; or perhaps better, of ultimate activities, that below the atom there are smaller units, the hypothetical electrons, which may be characterized as centers of force, and that they are due to condensation which produces the phenomena of gravitation. All further phenomena of physics and chemistry would have to be explained as the result of these elementary actions.

Formerly thinkers were inclined to see infinity all around. They thought of the atomic structure not only as infinitesimally small, but also as truly infinite; the molecules being analyzable into atoms and the atoms again into still smaller units, say into electrons or monads, and that the monads were again compounds of monadules and so forth—all this being argued on the poetic notion that

"Great fleas have little fleas
Upon their backs to bite 'em,
And little fleas have lesser fleas,
And so *ad infinitum.*"

The molecule is a kind of planetary system, with atoms as satellites, so is the atom with its circling electrons; why should not the electron be of the same construction and why should not the component parts of the electron be assumed to be made after the same pattern world without end? On the other hand our solar system is one among uncountably many others of the Milky Way; and the Milky Way in its turn is one universe of an enormously larger system of many Milky Ways. This is the conclusion which astronomy has deduced from actual facts. Why then should not this in our opinion enormous system of the many Milky Ways be only a tiny item in a still larger sys-

tem, and why should we not be justified in the assumption that we are confronted with an infinite vista into both directions toward the infinitely small and the infinitely great?

This notion has been brought out in the second quatrain which reads:

> "And the great fleas themselves in turn
> Have greater fleas to go on,
> While these again have greater still,
> And greater still and so on."

A vista into infinitudes, going out into the infinitely small and the infinitely great, now seems to become untenable, and definite limits loom up, which condition, so it seems to us, would reveal, not a bottomless and undefinable relativity but a definite world of an objective reality with definite interrelations and limits. If there are definite limits in either direction we may fairly well assume that they are in both directions. Further, if the universe is definite in its space relation, it should also be definitely limited in time. The world may have originated in an immeasurable ocean of uniformities as a definite commotion and may terminate again in a general dissolution by dissipation. If such be the case the relativity principle would not apply to the whole. Relativity would mean the interrelationship of all things, but the whole as a whole would be of a definite particularity with definite boundaries while the constitution of the world would exhibit a structure of extremely tiny ultimate units of a determinably definite size, endowed with a definite velocity and at every given point of a definite form of motion.

While the totality of existence, the sum total of our Milky Ways, appears to have had a beginning and may after the lapse of immeasurable ages come again to an end, we do not deem it excluded that the same process of world-formation may start again, as it probably was

repeated long before the origin of this our present universe. While thus everything existent, even the ether itself in its totality, would have to be regarded as particular and concrete with definite boundaries and as being limited to a definite time both in its beginning and in its end, there would after all loom up in the background of this world an infinitude of space, an eternity of time and an unfathomable wealth of potentialities as to new formations which in spite of all the light which the most advanced science will ever shed on the world problem will keep this great All of existence with its inexhaustible resources and its mysterious order an object of constant wonder and awe.

The relativity problem as such is a philosophical problem, but the relativity physicists have made a physical problem of it, and the philosophical problem of relativity is not a new problem, it is as old as science; it is only the lack of philosophical training which has led to the enunciation of some baffling paradoxes which if they were true would make objective science impossible, for they would abolish definiteness of any kind and do away with objectivity. And strange to say, claims of this kind are upheld on the ground of experiments which tend to establish the existence of an absolute, or as we would prefer to say, of some ultimate, which would prove that our experience does not float as a local tangle in an endless infinitude, but that there is a beginning and end, and also a boundary of all concrete reality at every definitely given moment. No mysticism is needed. Infinitude and eternality are potentialities, not actualities. They are vistas of what may be, not what is. They constitute the inexhaustible wealth of nature and of life without robbing science of its validity.

There is a tendency in mankind to think of the present moment as the climax of the past, which ushers in a new era by being an unprecedented and unique start. Every new generation passes through such a period of self-sufficiency

and of an intoxication with their own incomparable selfhood. The old problems seem new to them, and trying to formulate them in an original way, they applaud their own mistakes as something extraordinary and wonderful. Goethe characterizes this tendency in the young graduate who has just taken his degree of Bachelor (See *Faust,* Second Part, Act II) where this young man vents his ambitious conceit in these words:

"This is Youth's noblest calling and most fit!
The world was not, ere I created it;
The sun I drew from out the orient sea;
The moon began her changeful course with me;
The Day put on her shining robes, to greet me;
The Earth grew green, and burst in flower to meet me,
And when I beckoned, from the primal night
The stars unveiled their splendors to my sight.
Who, save myself, to you deliverance brought
From commonplaces of restricted thought?
I, proud and free, even as dictates my mind,
Follow with joy the inward light I find,
And speed along, in mine own ecstasy,
Darkness behind, and Glory leading me!"

It is apparent that the relativity physicists confront an important problem, but they have not succeeded in solving it; they have not even as yet properly formulated the question and their propositions are still in a state of fermentation. It is difficult to say what will come of it. It is to be hoped, however, that the movement will follow the usual course of mental growth. The relativists will drop their extravagant claims, they will mature the truth which they grope after and will at last formulate it into clear statements so as to justify the prophecy of Mephistopheles, who comments upon the proud words of the young Bachelor thus:

"Go hence, magnificent Original!—
What grief on thee would insight cast!
Who can think wise or stupid things at all,
That were not thought already in the Past?

Yet even from him we're not in special peril;
He will, ere long, to other thoughts incline:
The must may foam absurdly in the barrel,
Nathless it turns at last to wine."

THE PRINCIPLE OF RELATIVITY AS A PHASE IN THE DEVELOPMENT OF SCIENCE.

The principle of relativity made its appearance with great pretensions, and upset not a little the scientific world by its claim to antiquate the traditionally classical basis of physics, of astronomy, of mathematics, and of the other natural sciences. It affects especially the commonly accepted theory of the ether, and even the current views of space and time, which have hitherto proved serviceable. The entire realm of science was almost panic stricken for scientists seemed to have lost the *terra firma* under their feet; they felt as if they were sinking into a bottomless abyss and were left without a standing place in the whirl of a universal flux. Physicists of former date might take the movement for a joke, and many conservative thinkers find a good deal of humor in it, but the relativists are quite serious and are aware of the gravity of the consequences of their subversive work.

However, the new conception sailing under the flag of the principle of relativity which has been so noisily advanced to replace the old notions, does not prove quite satisfactory and presents too many difficulties to be acceptable to the average mind. It consists mainly of contradictory and mystifying statements commonly called "the paradoxes of relativity," and these statements have been praised or ridiculed, accepted or rejected, by enthusiastic adherents or obdurate adversaries, so that we have a state of things not unlike the rise of a new religious creed as it sets out to conquer the world. The names of Einstein, Lorentz, Minkowski, are the stars of first magnitude among the founders of the new world-conception. Their arguments, mathe-

matically well excogitated and worked out with subtile exactness, seem to carry everything before them, and we are not prepared to say that their contentions are wrong. Their propositions decidedly contain truths of great importance, referring mainly to calculations of minute precision in complicated phenomena. Yet common sense rebels against them and would not be convinced. *Prima facie* the new doctrine seems *ingeniosius quam verius*; it is ingeniously contrived but there is a hitch in it.

We have endeavored to show that the paradoxes have merely an appearance of contradictoriness; that they can be explained as slightly misstated conclusions and so the relativists could have avoided mystification. If physicists had borne in mind that as a matter of course all determinations and calculations of measurement require a reference point which remains unaltered in its relations to other points within the field of observation and is assumed to be stable, the problem would never have arisen. The new cases of unstable reference points which make their first appearance in 1727 with Bradley's investigations would simply have demanded a corrective without in the least upsetting the traditional view, and this will after all be the outcome of the new movement. It is to be anticipated that in the long run the paradoxical features of relativism will disappear, and when the results of the new propositions will be formulated without ado in sober consistency, it will be found that they only modify the old traditional physics and astronomy under certain specially complicated circumstances, particularly when the place of an observer while making his observations possesses a motion of its own affecting the motions under observation.

It almost seems as if the entire proposition of the principle of relativity might as well have been abandoned because the relativity of motion as well as of space, the field of motion, was not unknown to earlier physicists. Is not

relativity the nature of space? Denunciations to the effect that earlier physicists and astronomers had believed in an absolute space are really a misconstruction of their views, for what Newton called absolute space did not involve a denial of the conditions about which the relativists have troubled their minds.

As a symptom of sobering down, we mention an article of Edward V. Huntington which appeared in the *Philosophical Magazine* for April, 1912, under the title "A New Approach to the Theory of Relativity," pages 494 to 513. After explaining experiments with synchronized clocks, Professor Huntington says on page 507 as follows:

"These are the famous paradoxes of the theory of relativity which are often cited as proof of the assertion that the theory of relativity is incompatible with our ordinary ideas of time and space, but which here appear as necessary consequences of perfectly natural and reasonable conventions for setting clocks and laying out coordinates."

Further Professor Huntington takes away the mystery from some other propositions of the relativity principle. He concludes that "thus all the transformation equations used in theorem 1, are obtained by an entirely natural and elementary method."

The question then arises, Has the appearance of the principle of relativity done nothing to promote science, or has it even been a mistake? And we say in answer that the principle of relativity might have approached its problem in a more conservative way simply by bearing in mind that former physicists were perfectly aware of the fact that whenever they made measurements they laid down a point of reference for their calculations. This reference point must share the motion of the phenomena observed, or at least the difference must (for the purpose in view) be a negligible factor. We know very well, and all scientists of former generations also knew, that the stability of

our reference point is a fiction, but without making this fiction our calculations would sink into the bottomless.

While the geometer calculates distances on earth, he knows very well that the reference point from which he starts is not absolutely stable but moves around the sun with the entire earth, including the objects and the distances to be measured. When astronomers took measurements of the stars they knew very well that their own telescopes were moving along through space with the velocity of the earth under their feet, but for the purpose in view this movement was a negligible factor. Both physicists and astronomers of former times took it as a matter of course that we know of no point of absolute rest, that everything is relative, and that thus there is no absolute space in which their systems of measurement held a definite and invariable position. In this sense the principle of relativity is not quite so new as its enthusiastic adherents frequently claim. The contradictions and paradoxes are merely on the surface, and there is little hope of replacing the old orthodox mechanics which will even in the times to come hold good for all the usual commonplace fields of observation.

For all that, we do not mean to belittle the principle of relativity; the new method has its advantages, and in certain spheres it will find its application. What the relativists have accomplished may be comparable to the invention of a micrometer which proves very useful in making minute measurements hopelessly out of reach of the coarser instruments used in daily practical life. But as the micrometer will not abolish the usefulness of the yardstick, so relativist considerations will not upset the commonplace view of traditional mechanics. There is no contradiction between the two, if only we rightly understand the philosophical basis of the ordinary methods of measurement with their indispensable fiction of laying down a reference

point and ignoring the negligible factors of changes that take place while observations are being made, changes that may affect size, time and distance. The need of an indispensable reference point, the assumption of which is always a fiction made for the purpose in view, will be felt no less in the more complicated considerations which have prompted the rise of the new mechanics of the relativists.

In order to satisfy the demand of the conditions to which relativists have devoted their special attention, we have simply to bear in mind that the assumption of reference points is absolutely indispensable for any kind of measurement.

We may therefore say that the relativists have proposed their new theory as new in neglect of comprehending the philosophical basis of the science of measurements. We may grant that in the traditional treatment of kindred problems it was perhaps ignored, yet we trust that it was tacitly assumed.

The principle of relativity is therefore not useless, for it is serviceable in a field where more complicated interrelations have to be observed; but if the relativists dig deeper they will find common ground with their predecessors in the philosophical basis of the theory of measurement. As to the ether we must consider how little we know about its nature, and it seems premature either flatly to deny its existence, or to affirm doubtful qualities of it, or to make bold *a priori* statements as to its motions with reference to the motions of matter. Here experiments alone, like those made by Professor Michelson, will be decisive. Finally whatever difficulties may still present themselves, we may be assured that all of them will find a satisfactory solution without upsetting the foundations of our scientific world-conception.

CONCLUSION.

The theoretical problem of the principle of relativity has nothing to do with practical difficulties which are questions of fact. As the paradoxes disappear the theoretical problems are solved, while the practical difficulties must be overcome by experiment.

At the present state of our knowledge it would be fantastical to suggest a solution of the physical problems connected with the relativity movement, and we must leave the discussion of them to the future, for ere we can approach a solution we must know much more about the ultimate constituents of matter.

Who will furnish the key to the lock of the closed door at which the relativity physicists are knocking?

The details of the physical problems and their solution have only a slight interest for philosophy. The philosopher, however, expects that the physicist's solutions shall be consistent and that our scientific world-conception shall tolerate no contradictions.

If we consider the all-importance of form and the enormous significance which the formal sciences possess, we are inclined to regard the philosophy of relativity as a synonym and parallel development of the philosophy of science — the philosophy of form. But before we can definitely say so, we would expect the relativists to work out their philosophical substructure in a conservative way, to rid themselves of their paradoxical propositions, give up false pretensions to originality, recognize the past traditions of science, and rather than abandon the past, join their cause to the legitimate progress that follows from the tendencies, the ideals and aspirations of the established sciences.

We do not deny the relativity of all existence throughout and without exception, and in this sense we believe in

the principle of relativity, but we still cling to the old scientific ideal of objectivity and we can not see that the relativity principle as frequently enunciated by the relativists is well established.

The great question before the world of thinkers is this: Is it possible to construct a philosophy of science? The author of this essay has answered this question in the affirmative, and has worked in this field for fully a quarter of a century. He has worked out the details of a philosophy of science, and has submitted to the world in both *The Open Court* and *The Monist* his answers to the several philosophical questions. These questions are: the nature of the soul; the origin of sentiency and of thought; the nature of reason, especially in its origin and in its relation to language, the mechanism with which reason manifests itself; the nature of ethics and the foundation of morality as it is found in the laws of the objective world; the significance of the God-conception as the authority of conduct, as the ideal of right and wrong, as the standard of truth and error, as the object of devotion, of gratitude, of reverence mainly as the factor which determines good and evil. All these questions are not beyond the scope of scientific inquiry and in the philosophy of science definite solutions are propounded which, though based on radical principles of unbiased thought, lead to a justification of the historical growth of religion and science.

The whole scope of existence as it presents itself in human experience can become an object of scientific inquiry, and all scientific problems admit ultimately of a definite solution without equivocation or prevarication, yet at the same time science is only one attitude among several others from which the world can be confronted. The noetic conception is the ideal of understanding the world in its pure objectivity represented in mental terms to the exclusion of sentimental subjectivity. But man is not a child

of reason only. He is also endowed with sentiments, with will and with artistic tendencies. While the scientific world-conception is absolutely indispensable for the man of thought who works for a constant elevation of mankind upon a higher level, we must at the same time recognize the rights of the large masses who naturally are non-scientific and are swayed by sentiment, by devotion, by art, by ethical aspirations, by a religious comprehension of life; and thus we see in artistic and religious conceptions ways of treating the world problem which are by no means unjustified and ought not to be repudiated on the ground that they are non-scientific, sometimes unscientific, or even anti-scientific and purely sentimental. Religious cosmogonies, ecclesiastical ceremonies, religio-poetical fictions possess values of their own which can not and should not be measured by the standards of scientific method. The mystic also has his right to confront the world with his emotions and visions. Nevertheless, even here the philosophy of science will be capable of investigating various products of these tendencies and has a right to evaluate their truth or untruth by tracing the meaning of allegorical poetry as well as the wholesomeness of ethical attitudes which they encourage. In this way the philosophy of science as worked out by the present writer has by no means been narrow but has granted a free scope to all legitimate tendencies of the human mind, and if the philosophy of science has been properly understood, leaders of thought in the movements of pragmatism, relativism, Bergsonianism and other modern tendencies, would have been able to avoid at least some of their aberrations, and could have devoted their energies to efforts in the right direction. At any rate they would have been better understood; instead of being classified with philosophy, they would more properly have been regarded as a new species of poetry, or as literary ebullitions. Such they are; as such they possess value. They are not philos-

ophy, certainly not philosophy in the strict sense of the word; they are not scientific world-conceptions.

It may appear strange to class the movement which proclaims the principle of relativity in the same category with pragmatism and other antiscientific tendencies. We do so because the relativists have much in common with pragmatists, because both cancel the ideal of objectivity, both identify truth with the subjective conception of the real or with the observer's statement of facts. They identify size with result of measurement and think that the traditional view of truth is an error.

We do not overlook the fact that the relativists are of a highly intellectual type and employ scientific methods, but their aim is after all a denial of the old ideal of science, of the objectivity of truth, and of clearness of thought. All this is surrendered for the sake of a purely subjective simplification of statement which recommends itself in their own specialty. Certainly there is a great difference between relativists and pragmatists, but we recognize in both a subjectivist tendency and a subjectivist aim. Neither of them feel the need of approximating objectivity and both indulge in ideal constructions, both build air castles, the former of mathematical fiction, the latter of philosophical poetry.

All these modern anti-scientific isms may have originated through the one-sided tendencies of a misapplied scientism or even through the lack of comprehension of the principles and the significance of science among naturalists. These isms emphasize therefore certain contentions which have a nucleus of truth, by insisting on the rights of sentiment though they go too far when attacking science itself and claiming a superiority for unscientific sentiment over clear and methodical thought.

There is no question that all these modern movements try each in its own way to satisfy legitimate tendencies, but

in doing so they have mostly gone astray; partly they misunderstand their own aspirations, partly they lack sufficient depth of comprehension and width of horizon in encompassing the whole realm of human endeavor.

We do not expect that in this partisan scramble of various prejudices, the whole world of thinkers can be induced to recognize the common ideal of philosophical thought, but we hope that there will be enough minds to understand the several movements, to appreciate them so far as their aspirations are legitimate, and to discover their weak points in which they stray away from the straight path that leads forward to a truer, deeper and a broader conception of the world.

APPENDIX.

[The theory of the relativity of time and space, which is at present uppermost in the minds of physicists, has come into the foreground mainly through the differences of measuring at large distances the time it takes light to reach the observer's eye which is further complicated by the motions of his own standpoint. This happened for the first time in the history of science in the year 1726 when Mr. Bradley discovered that the fixed stars possessed a definite and peculiar motion of their own which was due to the motion of the earth around the sun and depended on the time it takes the light to reach the earth.

This classical exposition of his experiments was published in the form of a letter sent to the *Phil. Trans.* (Vol. XXXIV, p. 637) and has naturally become quite inaccessible. There is probably only one complete file of the *Transactions* west of the Alleghanies, the fortunate possessor of which is the Chicago Public Library. Considering the rarity of this essay we deem it proper to republish it and render it accessible to our readers. We do not doubt the very way in which Mr. Bradley approaches the problem will throw much light on the principle of relativity. In fact this essay will prove sufficient to explain its far-reaching significance, the need of its invention and the limitations of its use. A consideration of the foundaton of this principle and the history of its origin will clear it of the mysticism with which its recent representations have surrounded its statements.—P. C.]

THE REV. JAMES BRADLEY ON THE MOTION OF THE FIXED STARS.[1]

A Letter from the Reverend Mr. James Bradley, Savilian Professor of Astronomy at Oxford, and F. R. S., to Dr. Edmond Halley Astronom. Reg. &c. giving an Account of a new discovered Motion of the Fix'd Stars.

SIR,

You having been pleased to express your Satisfaction with what I had an Opportunity some time ago, of telling you in Conversation, concerning some Observations, that were making by our late worthy and ingenious Friend, the honorable *Samuel Molyneux* Esquire, and which have since been continued and repeated by myself, in order to determine the *Parallax* of the *fixt Stars*; I shall now beg leave to lay before you a more particular Account of them.

Before I proceed to give you the History of the Observations themselves, it may be proper to let you know, that they were at first begun in hopes of verifying and confirming those, that *Dr. Hook* formerly communicated to the publick, which seemed to be attended with Circumstances that promised greater Exactness in them, than could be expected in any other, that had been made and published on the same Account. And as his Attempt was what principally gave Rise to this, so his Method in making the Observations was in some Measure that which *Mr. Molyneux* followed: For he made Choice of the same Star, and his Instrument was constructed upon almost the same Principles. But if it had not greatly exceeded the Doctor's in Exactness, we might yet have remained in great Uncertainty as to the *Parallax* of the *fixt Stars*; as you will perceive upon the Comparison of the two Experiments.

This indeed was chiefly owing to our curious Member, Mr. *George Graham*, to whom the Lovers of Astronomy are also not a little

[1] Reprinted from the *Philosophical Transactions* of 1727.

indebted for several other exact and well-contrived Instruments. The Necessity of such will scarce be disputed by those that have had any Experience in making Astronomical Observations; and the Inconsistency, which is to be met with among different Authors in their Attempts to determine small Angles, particularly the annual Parallax of the *fixt Stars,* may be a sufficient Proof of it to others. Their Disagreement indeed in this article is not now so much to be wondered at, since I doubt not, but it will appear very probable, that the Instruments commonly made use of by them, were liable to greater Errors than many times that Parallax will amount to.

The Success then of this Experiment evidently depending very much on the Accurateness of the Instrument that was principally to be taken Care of: In what Manner this was done, is not my present Purpose to tell you; but if from the Result of the Observations which I now send you, it shall be judged necessary to communicate to the Curious the Manner of making them, I may hereafter perhaps give them a particular Description, not only of Mr. *Molyneux's* Instrument but also of my own, which hath since been erected for the same Purpose and upon the like Principles, though it is somewhat different in its Construction, for a Reason you will meet with presently.

Mr. *Molyneux's Apparatus* was compleated and fitted for observing about the End of November 1725, and on the third Day of *December* following, the bright Star at the Head of *Draco* (marked ν by *Bayer*) was for the first Time observed, as it passed near the Zenith, and its Situation carefully taken with the Instrument. The like Observations were made on the 5th, 11th and 12th Days of the same Month, and there appearing no material Difference in the Place of the Star, a farther Repetition of them at this Season seemed needless, it being a Part of the Year, wherein no sensible Alteration of Parallax in this Star could be expected. It was chiefly therefore Curiosity that tempted me (being then at *Kew,* where the Instrument was fixed) to prepare for observing the Star on *December* 17th, when having adjusted the Instrument as usual, I perceived that it passed a little more Southerly this Day than when it was observed before. Not suspecting any other Cause of this Appearance, we first concluded, that it was owing to the Uncertainty of the Observations, and that either this or the foregoing were not so exact as we had before supposed; for which Reason we purposed to repeat the Observation again, in order to determine from whence this Difference proceeded; and upon doing it on *December* 20th, I

found that the Star passed still more Southerly than in the former Observations. This sensible Alteration the more surprized us, in that it was the contrary way from what it would have been, had it proceeded from an annual Parallax of the Star: But being now pretty well satisfied, that it could not be entirely owing to the want of Exactness in the Observations; and having no Notion of anything else, that could cause such an apparent Motion as this in the Star; we began to think that some Change in the Materials, &c. of the Instrument itself, might have occasioned it. Under these Apprehensions we remained some time, but being at length fully convinced, by several Trials, of the great Exactness of the Instrument, and finding by the gradual Increase of the Star's Distance from the Pole, that there must be some regular Cause that produced it; we took care to examine nicely, at the Time of each Observation, how much it was: and about the Beginning of *March* 1725, the Star was found to be 20″ more Southerly than at the Time of the first Observation. It now indeed seemed to have arrived at its utmost Limit Southward, because in several Trials made about this Time, no sensible Difference was observed in its Situation. By the Middle of *April,* it appeared to be returning back again towards the North; and about the beginning of *June,* it passed at the same Distance from the Zenith as it had done in *December* when it was first observed.

From the quick Alteration of this Star's Declination about this Time (it increasing a Second in three Days) it was concluded, that it would now proceed Northward, as it before had done Southward of its present Situation; and it happened as was conjectured: for the Star continued to move Northward till *September* following, when it again became stationary, being then near 20″ more Northerly than in *June,* and no less than 39″ more Northerly than it was in *March.* From *September* the Star returned towards the South, till it arrived in *December* to the same Situation it was in at that time twelve Months, allowing for the Difference of Declination on account of the Precession of the Equinox.

This was a sufficient Proof, that the Instrument had not been the Cause of this apparent Motion of the Star, and to find one adequate to such an Effect seemed a Difficulty. A Nutation of the Earth's Axis was one of the first things that offered itself upon this Occasion, but it was soon found to be insufficient; for though it might have accounted for the change of Declination in *ν Draconis* yet it would not at the same time agree with the Phaenomena in other Stars; particularly in a small one almost opposite in right

Ascension to *ν Draconis,* at about the same Distance from the North Pole of the Equator: For, though this Star seemed to move the same way, as a Nutation of the Earth's Axis would have made it, yet it changing its Declination but about half as much as *ν Draconis* in the same time (as appeared upon comparing the Observations of both made upon the same Days, at different Seasons of the Year) this plainly proved, that the apparent Motion of the Stars was not occasioned by a real Nutation, since if that had been the Cause, the Alteration in both Stars would have been near equal.

The great Regularity of the Observations left no room to doubt, but that there was some regular Cause that produced this unexpected Motion, which did not depend on the Uncertainty or Variety of the Seasons of the Year. Upon comparing the Observations with each other, it was discovered that in both the fore-mentioned Stars, the apparent Difference of Declination from the *Maxima,* was always nearly proportional to the versed Sine of the Sun's Distance from the Equinoctial Points. This was an Inducement to think, that the Cause, whatever it was, had some Relation to the Sun's Situation with respect to those Points. But not being able to frame any Hypothesis at that Time sufficient to solve all the Phænomena, and being very desirous to search a little farther into this Matter; I began to think of erecting an Instrument for myself at Wansted, that having it always at Hand, I might with the more Ease and Certainty, enquire into the Laws of this new Motion. The Consideration likewise of being able by another Instrument, to confirm the Truth of the Observations hitherto made with *Mr. Molyneux's,* was no small Inducement to me; but the Chief of all was, the Opportunity I should thereby have of trying, in what Manner other Stars were affected by the same Cause, whatever it was. For *Mr. Molyneux's* Instrument being originally designed for observing *ν Draconis* (in order as I said before, to try whether it had any sensible Parallax) was so contrived, as to be capable of but little Alteration in its Direction, not above seven or eight Minutes of a Degree; and there being few stars within half that Distance from the Zenith of *Kew,* bright enough to be well observed, he could not, with his Instrument, thoroughly examine how this Cause affected Stars differently situated with respect to the equinoctial and solstitial Points of the Ecliptick.

These Considerations determined me; and by the Contrivance and Direction of the same ingenious Person, Mr. Graham, my Instrument was fixed up *August* 19, 1727. As I had no convenient

Place where I could made use of so long a Telescope as *Mr. Molyneux's,* I contented myself with one of but little more than half the Length of his (viz. of about 12½ Feet, his being 24½) judging from the Experience which I had already had, that this Radius would be long enough to adjust the Instrument to a sufficient Degree of Exactness, and I have no reason since to change my Opinion: for from all the Trials I have yet made, I am very well satisfied, that when it is carefully rectified, its Situation may be securely depended upon to half a Second. As the Place where my Instrument was to be hung, in some Measure determined its Radius, so did it also the Length of the Arch, or Limb, on which the Divisions were made to adjust it: For the Arch could not conveniently be extended farther, than to reach to about 6¼° on each Side my Zenith. This indeed was sufficient, since it gave me an Opportunity of making Choice of several Stars, very different both in Magnitude and Situation; there being more than two hundred inserted in the *British* Catalogue, that may be observed with it. I needed not to have extended the Limb so far, but that I was willing to take in *Capella,* the only star of the first Magnitude that comes so near my Zenith.

My instrument being fixed, I immediately began to observe such Stars as I judged most proper to give me light into the Cause of the Motion already mentioned. There was Variety enough of small ones; and not less than twelve, that I could observe through all the Seasons of the Year; they being bright enough to be seen in the Day-time, when nearest the Sun. I had not been long observing, before I perceived, that the Notion we had before entertained of the Stars being farthest North and South, when the Sun was about the Equinoxes, was only true of those that were near the solstitial Colure: And after I had continued my Observations a few Months, I discovered, what I then apprehended to be a general Law, observed by all the Stars, viz. That each of them became stationary, or was farthest North or South, when they passed over my Zenith at six of the Clock, either in the Morning or Evening. I perceived likewise, that whatever Situation the Stars were in with respect to the cardinal Points of the Ecliptick, the apparent motion of every one tended the same Way, when they passed my instrument about the same Hour of the Day or Night; for they all moved Southward, while they passed in the Day, and Northward in the Night; so that each was farthest North, when it came about Six of the Clock in the Evening, and farther South, when it came about Six in the Morning.

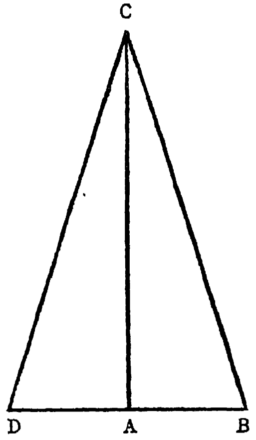
Though I have since discovered, that the *Maxima* in most of these Stars do not happen exactly when they come to my Instrument at those Hours, yet not being able at that time to prove the contrary, and supposing that they did, I endeavoured to find out what Proportion the greatest Alterations of Declination in different Stars bore to each other; it being very evident, that they did not all change their Declination equally. I have before taken notice, that it appeared from Mr. *Molyneux's* Observations, that ν Draconis altered its Declination about twice as much as the fore-mentioned small Star almost opposite to it; but examining the matter more particularly, I found that the greatest Alteration of Declination in these Stars, was at the Sine of the Latitude of each respectively. This made me suspect that there might be the like Proportion between the *Maxima* of other Stars; but finding, that the observations of some of them would not perfectly correspond with such an Hypothesis, and not knowing, whether the small Difference I met with, might not be owing to the Uncertainty and Error of the Observations, I deferred the farther examination into the Truth of this Hypothesis, till I should be furnished with a Series of Observations made in all Parts of the Year; which might enable me, not only to determine what Errors the Observations are liable to, or how far they may safely be depended upon; but also to judge, whether there had been any sensible Change in the Parts of the Instrument itself.

Upon these Considerations, I laid aside all Thoughts at that Time about the Cause of the fore-mentioned Phaenomena, hoping that I should the easier discover it, when I was better provided with proper Means to determine more precisely what they were.

When the Year was compleated, I began to examine and compare my Observations, and having pretty well satisfied myself as to the general Laws of the Phaenomena, I then endeavoured to find out the Cause, of them. I was already convinced, that the apparent Motion of the Stars, was not owing to a Nutation of the Earth's Axis. The next Thing that offered itself, was an Alteration in the Direction of the Plumb-line, with which the Instrument was constantly rectified; but this upon Trial proved insufficient. Then I considered what Refraction might do, but here also nothing satisfactory occurred. At last I conjectured, that all the Phaenomena hitherto mentioned, proceeded from the progressive Motion of Light and the Earth's annual Motion in its Orbit. For I perceived, that, if Light was propagated in Time, the apparent Place of a fixt Object would not be the same when the Eye is at Rest, as when it is

moving in any other Direction, than that of the Line passing through the Eye and Object; and that, when the Eye is moving in different Directions, the apparent Place of the Object would be different.

I considered this Matter in the following Manner. I imagined CA to be a Ray of Light, falling perpendicularly upon the Line BD; then if the Eye is at rest at A, the Object must appear in the Direction AC, whether Light be propagated in Time or in an Instant. But if the Eye is moving from B towards A, and Light is propagated in Time, with a Velocity that is to the Velocity of the Eye, as CA to BA; then Light moving from C to A, whilst the Eye moves from B to A, that Particle of it, by which the Object will be discerned, when the Eye in its Motion comes to A, is at C when the Eye is at B. Joining the Points B, C, I supposed the Line CB,



to be a Tube (inclined to the Line BD in the Angle DBC) of such a Diameter, as to admit of but one Particle of Light; then it was easy to conceive, that the Particle of Light at C (by which the object must be seen when the Eye, as it moves along, arrives at A) would pass through the Tube BC, if it is inclined to BD in the Angle DBC, and accompanies the Eye in its Motion from B to A; and that it could not come to the Eye, placed behind such a Tube, if it had any other Inclination to the Line BD. If instead of supposing CB so small a Tube, we imagine it to be the Axis of a larger; then for the same Reason, the Particle of Light at C, could not pass through that Axis, unless it is inclined to BD, in the Angle CBD. In like manner, if the Eye moved the contrary way, from D towards A, with the same Velocity; then the Tube must be inclined in the Angle

BDC. Although therefore the true or real Place of an Object is perpendicular to the Line in which the Eye is moving, yet the visible Place will not be so, since that, no doubt, must be in the Direction of the Tube; but the Difference between the true and apparent Place will be (*cæteris paribus*) greater or less, according to the different Proportion between the Velocity of Light and that of the Eye. So that if we could suppose that Light was propagated in an instant, then there would be no Difference between the real and visible Place of an Object, although the Eye were in Motion, for in that case, AC being infinite with Respect to AB, the Angle ACB (the Difference between the true and visible Place) vanishes. But if Light be propagated in Time (which I presume will readily be allowed by most of the Philosophers of this Age) then it is evident from the foregoing Considerations, that there will be always a Difference between the real and visible Place of an Object, unless the Eye is moving either directly towards or from the Object. And in all Cases, the Sine of the Difference between the real and visible Place of the Object, will be to the Sine of the visible Inclination of the Object to the Line in which the Eye is moving, as the Velocity of the Eye to the Velocity of Light.

If Light moved but 1000 times faster than the Eye, and an Object (supposed to be at an infinite Distance) was really placed perpendicularly over the Plain in which the Eye is moving, it follows from what hath been already said, that the apparent Place of such an Object will be always inclined to that Plain, in an Angle of $89° \; 56'\frac{1}{2}$; so that it will constantly appear $3'\frac{1}{2}$ from its true Place, and seem so much less inclined to the Plain, that way towards which the Eye tends. That is, if AC is to AB (or AD) as 1000 to one, the Angle ABC will be $89° \; 56'\frac{1}{2}$, and $ACB = 3'\frac{1}{2}$, and $BCD = 2ACB = 7'$. So that according to this Supposition, the visible or apparent Place of the Object will be altered $7'$, if the Direction of the Eye's Motion is at one time contrary to what it is at another.

If the Earth revolve round the Sun annually, and the Velocity of Light were to the Velocity of the Earth's Motion in its Orbit (which I will at present suppose to be a Circle) as 1000 to one; then tis easy to conceive, that a Star really placed in the very Pole of the Ecliptick, would, to an Eye carried along with the Earth, seem to change its Place continually, and (neglecting the small Difference on the Account of the Earth's diurnal Revolution on its Axis) would seem to describe a Circle round that Pole, every Way distant therefrom $3'\frac{1}{2}$. So that its Longitude would be varied through all the

Points of the Ecliptick every Year; but its Latitude would always remain the same. Its right Ascension would also change, and its Declination, according to the different Situation of the Sun in respect to the equinoctial Points; and its apparent Distance from the North Pole of the Equator would be 7′ less at the Autumnal, than at the vernal Equinox.

The greatest Alteration of the Place of a Star in the Pole of the Ecliptick (or which in Effect amounts to the same, the Proportion between the Velocity of Light and the Earth's Motion in its Orbit) being known; it will not be difficult to find what would be the Difference upon this Account, the Difference between the true and apparent Place of any other Star at any time; and on the contrary, the Difference between the true and apparent Place being given; the Proportion between the Velocity of Light and the Earth's Motion in its Orbit may be found.

As I only observed the apparent Difference of Declination of the Stars, I shall not now take any farther Notice in what manner such a Cause as I have here supposed would occasion an Alteration in their apparent Places in other Respects; but, supposing the Earth to move equally in a Circle, it may be gathered from what hath been already said, that a Star which is neither in the Pole nor Plain of the Ecliptick, will seem to describe about its true Place a Figure, insensibly different from an Ellipse, whose Transverse Axis is at Right-angle to the Circle of Longitude passing through the Star's true Place, and equal to the Diameter of the little Circle described by a Star (as was before supposed) in the Pole of the Ecliptick; and whose Conjugate Axis is to its Transverse Axis, as the *Sine* of the Star's latitude to the Radius. And allowing that a Star by its apparent Motion does exactly describe such an Ellipse, it will be found, that if A be the Angle of Position (or the Angle at the Star made by two great Circles drawn from it, thro' the Poles of the Ecliptick and Equator) and B be another Angle, whose Tangent is to the Tangent of A as Radius to the Sine of the Latitude of the Star; then B will be equal to the Difference of Longitude between the Sun and the Star, when the true and apparent Declination of the Star are the same. And if the Sun's Longitude in the Ecliptick be reckoned from that Point, wherein it is when this happens; then the Difference between the true and apparent Declination of the Star (on account of the Cause I am now considering) will be always, as the Sine of the Sun's Longitude from thence. It will likewise be found, that the greatest Difference of Declination that can be

between the true and apparent Place of the Star, will be to the Semi-Transverse Axis of the Ellipse (or to the Semi-diameter of the little Circle described by a Star in the Pole of the Ecliptick) as the Sine of A to the Sine of B.

If the Star hath North Latitude, the Time, when its true and apparent Declination are the same, is before the Sun comes in Conjunction with or Opposition to it, if its Longitude be in the first or last Quadrant (viz. in the ascending Semi-circle) of the Ecliptick; and after them, if in the descending Semi-circle; and it will appear nearest to the North Pole of the Equator, at the Time of that *Maximum* (or when the greatest Difference between the true and apparent Declination happens) which precedes the Sun's Conjunction with the Star.

These Particulars being sufficient for my present Purpose, I shall not detain you with the Recital of any more, or with any farther Explication of these. It may be time enough to enlarge more upon this Head, when I give a Description of the Instruments &c. if that be judged necessary to be done; and when I shall find, what I now advance, to be allowed of (as I flatter myself it will) as something more than a bare Hypothesis. I have purposely omitted some matters of no great Moment, and considerd the Earth as moving in a Circle, and not an Ellipse, to avoid too perplexed a *Calculus,* which after all the Trouble of it would not sensibly differ from that which I make use of, especially in those Consequences which I shall at present draw from the foregoing Hypothesis.

This being premised, I shall not proceed to determine from the observations, what the real Proportion is between the Velocity of Light and the Velocity of the Earth's annual Motion in its Orbit; upon Supposition that the Phaenomena before mentioned do depend upon the Causes I have here assigned. But I must first let you know, that in all the Observations hereafter mentioned, I have made an Allowance for the Change of the Star's Declination on Account of the Precession of the Equinox, upon Supposition that the Alteration from this Cause is proportional to the Time, and regular through all the Parts of the Year. I have deduced the real annual Alteration of Declination of each Star from the Observations themselves; and I the rather choose to depend upon them in this Article, because all which I have yet made, concur to prove, that the Stars near the Equinoctial Colure, change their Declination at this time $1''\frac{1}{2}$ or $2''$ in a Year more than they would do if the Precession was only $50''$, as is now generally supposed. I have likewise met with some small

Varieties in the Declination of other Stars in different Years, which do not seem to proceed from the same Cause, particularly in those that are near the solstitial Colure, which on the contrary have altered their Declination less than they ought, if the Precession was 50″. But whether these small Alterations proceed from a regular Cause, or are occasioned by any Change in the Materials &c. of my Instrument, I am not yet able fully to determine. However, I thought it might not be amiss just to mention to you how I have endeavoured to allow for them, though the Result would have been nearly the same, if I had not considered them at all. What that is, I will shew, first from the Observations of ν *Draconis,* which was found to be 39″ more Southerly in the Beginning of *March,* than in *September.*

From what hath been premised, it will appear that the greatest Alteration of the apparent Declination of ν *Draconis,* on account of the successive Propagation of Light, would be to the Diameter of the little Circle which a Star (as was before remarked) would seem to describe about the Pole of the Ecliptick as 39″ to 40″, 4. The half of this is the Angle ACB (as represented in the Fig.) This therefore being 20″, 2, AC will be to AB, that is, the Velocity of Light to the Velocity of the Eye (which in this Case may be supposed the same as the Velocity of the Earth's annual Motion in its Orbit) as 10210 to One, from whence it would follow, that Light moves, or is propagated as far as from the Sun to the Earth in 8′ and 12″.

It is well known, that Mr. *Romer*, who first attempted to account for an apparent Inequality in the Times of the Eclipses of *Jupiter's* Satellites, by the Hypothesis of the progressive Motion of Light, supposed that it spent about 11 Minutes of Time in its Passage from the Sun to us: but it hath since been concluded by others from the like Eclipses, that it is propagated as far in about 7 Minutes. The Velocity of Light therefore deduced from the foregoing Hypothesis, is as it were a Mean betwixt what had at different times been determined from the Eclipses of *Jupiter's* Satellites.

These different Methods of finding the Velocity of Light thus agreeing in the Result, we may reasonably conclude, not only that these *Phaenomena* are owing to the Causes to which they have been ascribed; but also, that Light is propagated (in the same Medium) with the same Velocity after it hath been reflected as before; for this will be the Consequence, if we allow that the Light of the Sun is propagated with the same Velocity, before it is reflected, as the Light of the *fixt Stars.* And I imagine this will

scarce be questioned, if it can be made appear that the Velocity of the Light of all the *fixt Stars* is equal, and that their Light moves or is propagated through equal Spaces in equal Times, at all Distances from them: both which points (as I apprehend) are sufficiently proved from the apparent alteration of the Declination of Stars of different Lustre; for that is not sensibly different in such Stars as seem near together, though they appear of very different Magnitudes. And whatever their Situations are (if I proceed according to the foregoing Hypothesis) I find the same Velocity of Light from my Observations of small Stars of the fifth or sixth, as from those of the second and third Magnitude, which in all Probability are placed at very different Distances from us. The small Star, for Example, before spoken of, that is almost opposite to ν *Draconis* (being the 35th *Camelopard. Hevelii* in Mr. *Flamsteed's* Catalogue) was 19″ more Northerly about the Beginning of *March* than in *September.* Whence I conclude, according to my Hypothesis, that the Diameter of the little Circle described by a Star in the Pole of the Ecliptick would be 40″, 2.

The last Star of the great Bear's tail of the 2d Magnitude (marked η by *Bayer*) was 36″ more Southerly about the Middle of *January* than in *July.* Hence the *Maximum,* or greatest Alteration of Declination of a Star in the Pole of the Ecliptick would be 40″, 4, exactly the same as was before found from the Observations of ν *Draconis.*

The Star of the 5th magnitude in the Head of *Perseus* marked τ by *Bayer,* was 25″ more Northerly about the End of *December* than on the 29th of *July* following. Hence the *Maximum* would be 41″. This Star is not bright enough to be seen as it passes over my Zenith about the End of *June,* when it should be according to the Hypothesis farthest South. But because I can more certainly depend upon the greatest Alteration of Declination of those Stars, which I have frequently observed about the Times when they become stationary, with respect to the Motion I am now considering; I will set down a few more Instances of such, from which you may be able to judge how near it may be possible from these Observations, to determine with what Velocity Light is propagated.

α *Persei Bayero* was 23″ more Northerly at the beginning of *January* than in *July.* Hence the *Maximum* would be 40″, 2. α *Cassiopeæ* was 34″ more Northerly about the End of *December* than in *June.* Hence the *Maximum* would be 40″, 8. β *Draconis* was 39″ more Northerly in the beginning of *September* than in *March*;

hence the *Maximum* would be 40″, 2. *Capella* was about 16″ more Southerly in *August* than in *Feb.*; hence the *Maximum* would be about 40″. But this Star being farther from my Zenith than those I have before made use of, I cannot so well depend upon my Observations of it, as of the others; because I meet with some small Alterations of its Declination that do not seem to proceed from the Cause I am now considering.

I have compared the Observations of several other Stars, and they all conspire to prove that the *Maximum* is about 40″ or 41″. I will therefore suppose that it is 40″½ or (which amounts to the same) that Light moves, or is propagated as far as from the Sun to us in 8′ 13″. The near Agreement which I met with among my Observations induces me to think, that the *Maximum* (as I have here fixed it) cannot differ so much as a Second from the Truth, and therefore it is probable that the Time which Light spends in passing from the Sun to us, may be determined by these Observations within 5″ or 10″; which is such a degree of exactness as we can never hope to attain from the Eclipses of *Jupiter's* Satellites.

Having thus found the Maximum, or what the greatest Alternation of Declination would be in a Star placed in the Pole of the Ecliptick, I will now deduce from it (according to the foregoing Hypothesis) the Alteration of Declination in one or two Stars, at such times as they were actually observed, in order to see how the Hypothesis will correspond with the *Phænomena* through all the Parts of the Year.

It would be too tedious to set down the whole Series of my Observations; I will therefore make Choice only of such as are most proper for my present Purpose, and will begin with those of *ν Draconis.*

This Star appeared farthest North about *September* 7th, 1727, as it ought to have done according to my Hypothesis. The following Table shews how much more Southerly the star was found to be by Observation in several Parts of the Year, and how much more Southerly it ought to be according to the Hypothesis.

Hence it appears, that the Hypothesis corresponds with the Observations of this Star through all Parts of the Year; for the small Differences between them seem to arise from the Uncertainty of the Observations, which is occasioned (as I imagine) chiefly by the tremulous or undulating Motion of the Air, and of the Vapours in it; which causes the Stars sometimes to dance to and fro, so

1727 D.	THE DIFFERENCE OF DECLINATION BY OBSERVATION	THE DIFFERENCE OF DECLINATION BY THE HYPOTHESIS	1728 D.	THE DIFFERENCE OF DECLINATION BY OBSERVATION	THE DIFFERENCE OF DECLINATION BY THE HYPOTHESIS
Oct. 20	4½	4½	Mar. 24	37	38
Nov. 17	11½	12	April 6	36	36½
Dec. 6	17½	18½	May 6	28½	29½
Dec. 28	25	26	June 5	18½	20
1728			June 15	17½	17
Jan. 24	34	34	July 3	11½	11½
Feb. 10	38	37	Aug. 2	4	4
Mar. 7	39	39	Sept. 6	0	0

much that it is difficult to judge when they are exactly on the Middle of the Wire that is fixed in the common Focus of the Glasses of the Telescope.

I must confess to you, that the Agreement of the Observations with each other, as well as with the Hypothesis, is much greater than I expected to find, before I had compared them; and it may possibly be thought to be too great, by those who have been used to Astronomical Observations, and know how difficult it is to make such as are in all respects exact. But if it would be any Satisfaction to such Persons (till I have an Opportunity of describing my Instrument and the manner of using it) I could assure them, that in above 70 Observations which I made of this Star in a Year, there is but one (and that is noted as very dubious on account of Clouds) which differs from the foregoing Hypothesis more than 2″, and this does not differ 3″.

This therefore being the Fact, I cannot but think it very probable, that the *Phænomena* proceed from the Cause I have assigned, since the foregoing Observations make it sufficiently evident, that the Effect of the real Cause, whatever it is, varies in this Star, in the same Proportion that it ought according to the Hypothesis.

But least *ν Draconis* may be thought not so proper to shew the proportion, in which the apparent alteration of Declination is increased or diminished, as those Stars which lie near the Equinoctial Colure: I will give you also the Comparison between the Hypoth-

esis and the Observations of η *Ursæ Majoris,* that which was farthest South about the 17th Day of *January* 1728, agreable tó the Hypothesis. The following Table shews how much more Northerly it was found by Observation in several Parts of the Year, and also what the Difference should have been according to the Hypothesis.

1727 d.	THE DIFFERENCE OF DECLINATION BY OBSERVATION	THE DIFFERENCE OF DECLINATION BY THE HYPOTHESIS	1728 d.	THE DIFFERENCE OF DECLINATION BY OBSERVATION	THE DIFFERENCE OF DECLINATION BY THE HYPOTHESIS
Sept. 14	29½	28½	April 16	18½	18
Sept. 24	24½	25½	May 5	24½	23½
Oct. 16	19½	19½	June 5	32	31½
Nov. 11	11½	10½	June 25	35	34½
Dec. 14	4	3	July 17	36	36
1728			Aug. 2	35	35½
Feb. 17	2	3	Sept. 20	26½	26½
Mar. 21	11½	10½			

I find upon Examination, that the Hypothesis agrees altogether as exactly with the Observations of this Star, as the former; for in about 50 that were made of it in a Year, I do not meet with a Difference of so much as 2″, except in one, which is mark'd as doubtful on Account of the Undulation of the Air &c. And this does not differ 3″ from the Hypothesis.

The agreement between the Hypothesis and the Observations of this Star is the more to be reguarded, since it proves that the Alteration of Declination, on account of the Precession of the Equinox, is (as I before supposed) regular thro' all Parts of the Years; so far at least, as not to occasion a Difference great enough to be discovered with this Instrument. It likewise proves the other part of my former Supposition, viz. that the annual Alteration of Declination in Stars near the Equinoctial Colure, is at this Time greater than a Precession of 50″ would occasion: for this Star was 20″ more Southerly in *September* 1728, than in *September* 1727, that is, about 2″ more than it would have been, if the Precession was but 50″. But I may hereafter, perhaps, be better able to determine this Point,

from my Observations of those Stars that lie near the Equinoctial Colure, at about the same Distance from the North Pole of the Equator, and nearly opposite in right Ascension.

I think it needless to give you the Comparison between the Hypothesis and the Observations of any more Stars; since the Agreement in the foregoing is a kind of Demonstration (whether it be allowed that I have discovered the real Cause of the *Phænomena* or not;) that the Hypothesis gives at least the true Law of the Variation of Declination in different Stars, with Respect to their different Situations and Aspects with the Sun. And if this is the Case, it must be granted, that the Parallax of the fixt Stars is much smaller, than hath been hitherto supposed by those who have pretended to deduce it from their Observations. I believe, that I may venture to say, that in either of the two Stars, last mentioned, it does not amount to 2″. I am of Opinion, that if it were 1″, I should have perceived it, in the great number of Observations that I made especially of *ν Draconis*; which agreeing with the Hypothesis (without allowing anything for Parallax) nearly as well when the Sun was in Conjunction with, as in Opposition to, this Star, it seems very probable that the Parallax of it is not so great as one single Second; and Consequently that it is above 400000 times farther from us than the Sun.

There appearing therefore after all, no sensible Parallax in the fixt Stars, the *Anti-Copernicans* have still room on that Account, to object against the Motion of the Earth; and they may have (if they please) a much greater objection against the Hypothesis, by which I have endeavoured to solve the fore-mentioned *Phænomena*; by denying the progressive Motion of Light, as well as that of the Earth.

But as I do not apprehend, that either of these Postulates will be denied me by the Generality of the Astronomers and Philosophers of the present Age; so I shall not doubt of obtaining their Assent to the Consequences which I have deduced from them; if they are such as have the Approbation of so great a Judge of them as Yourself. I am

Sir, Your most Obedient
Humble Servant
J. BRADLEY.

POSTSCRIPT.

As to the Observations of *Dr. Hook,* I must own to you, that before Mr. *Molyneux's* Instrument was erected, I had no small

opinion of their Correctness; the Length of his Telescope and the Care he pretends to have taken in making them exact, having been strong Inducements with me to think them so. And Since I have been convinced both from Mr. *Molyneux's* Observations and my own, that the Doctor's are really very far from being either exact or agreeable to the *Phænomena*; I am greatly at a loss how to account for it. I cannot well conceive that an Instrument of the Length of 36 Feet, constructed in the Manner he describes his, could have been liable to an Error of near 30″ (which was doubtless the Case) if rectified with so much Care as he represents.

The Observations of Mr. *Flamsteed* of the different Distances of the Pole Star from the Pole at different Times of the Year, which were through Mistake looked upon by some as a Proof of the annual *Parallax* of it, seem to have been made with much greater Care than those of *Dr. Hook*. For though they do not all exactly correspond with each other, yet from the whole Mr. *Flamsteed* concluded that the Star was 35″ 40″ or 45″ nearer the Pole in *December* than in *May* or *July*: and according to my Hypothesis it ought to appear 40″ nearer in *December* than in *June*. The Agreement therefore of the Observations with the Hypothesis is greater than could reasonably be expected, considering the *Radius* of the Instrument, and the Manner in which it was constructed.